Target Tracking in the Framework of Possibility Theory

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Uncertainty is the essential attribute of target tracking. It needs to be included in mathematical modelling of all aspects of a target tracking system, such as target dynamics, the target birth/disappearance process, measurement characteristics (probability of detection, characterisation of false alarms, and measurement noise), and prior contextual information (domain knowledge, such as maps and corridors, and historical data). The vast majority of target tracking algorithms are formulated as Bayesian inference problems, with uncertainty characterised by probabilistic models.

In practical real-world applications of target tracking, however, the specification of these (precise) probabilistic models for all uncertain aspects of this complex problem is often difficult. For example, the probability of detection as a function of range would depend on many unknown factors (e.g., environmental conditions, object size, and its reflective characteristics); hence, a precise trustworthy model would be almost impossible to put forward. This inherent misspecification often implies that several heuristics have to be introduced to compensate for the discrepancies between the model and the real data. It also relates to the common aphorism in statistics: all models are wrong, but some are useful.

In technical terms, probability theory deals with only one aspect of uncertainty involved in modelling complex systems (such as target tracking): uncertainty due to randomness [1]. This relates to the *known unknowns* feature of a model, where the probability function is known but the actual realisation is random (and hence unknown). Arguably, there is another layer of uncertainty involved in modelling, the *unknown unknowns* factor, often referred to as epistemic uncertainty. The existence of epistemic uncertainty has been the motivation behind several recent theories for quantitative modelling of uncertainty, such as possibility theory, Dempster-Shafer theory, and imprecise probability theory [2]. The focus in this article is on possibility theory, because (1) the standard probabilistic concepts can be (relatively easily) extended to this context and (2) at present, the last two aforementioned theories are primarily developed for and limited to discrete state spaces.

THEORETICAL FOUNDATIONS

UNCERTAIN VARIABLE

The concept of *uncertain variable,* in the adopted framework of possibility theory, plays the same role as a random variable in probability theory. The main difference is that the quantities of interest are not random but simply unknown, and our aim is to infer their true values out of a set of possible values. The theoretical basis of this approach can be found in [3], [4]. Briefly, the uncertain variable is a function $\mathbf{X}:\Omega \to \mathbb{X}$, where Ω is the sample space and $\mathbb{X}$ is the state space (the space where the quantity of interest lives). Our current knowledge about $\mathbf{X}$ can be encoded in a function $\pi_X : \mathbb{X} \to [0,1]$, such that $\pi_X(\mathbf{x})$ is the possibility (credibility) for the event $\mathbf{X} = \mathbf{x}$. Function π_X is not a density function; it is referred to as a possibility function, being the primitive object of possibility theory [5]. It can be seen as a membership function that determines the fuzzy restriction of minimal specificity (in the sense that any hypothesis not known to be impossible cannot be ruled out) about $\mathbf{x}$ [6]. Normalisation of π_X is $\sup_{\mathbf{x}\in\mathbb{X}} \pi_X(\mathbf{x}) = 1$ if $\mathbb{X}$ is uncountable and $\max_{\mathbf{x}\in\mathbb{X}} \pi_X(\mathbf{x}) = 1$ if $\mathbb{X}$ is countable.

The objective is to carry out inference on dynamical systems in a manner analogous to the Bayesian formulation. Then, it is natural to consider sequences $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_k \equiv \mathbf{x}_{1:k}$ of uncertain variables, with k being a discrete-time index and $\mathbf{x}_k$ representing the state of the target of interest in $\mathbb{X}$ at time k. Such a sequence of uncertain variables is an uncertain process (chain) [4]. An uncertain process is Markovian if $\pi_{k|1:k-1}(\mathbf{x}_k \mid \mathbf{x}_{1:k-1}) = \pi_{k|k-1}(\mathbf{x}_k \mid \mathbf{x}_{k-1})$, for any $\mathbf{x}_1,\ldots,\mathbf{x}_k \in \mathbb{X}$.

NONLINEAR FILTERING

Nonlinear filtering in the framework of possibility theory is formulated next. Let the target dynamics be specified by the transition possibility function $\rho_{k|k-1}(\mathbf{x} \mid \mathbf{x}')$, which specifies the uncertain evolution of the state from time $k - 1$ to time k. Let the uncertain relationship between the target-originated measurement $\mathbf{z} \in \mathbb{Z}$ and the (hidden) target state $\mathbf{x}$ at time k be specified by the likelihood function $g_k(\mathbf{z}|\mathbf{x})$, expressed as a possibility function. Here, $\mathbb{Z}$ is the measurement space. Given the dynamics model $\rho_{k|k-1}(\mathbf{x}|\mathbf{x}')$ and the measurement model $g_k(\mathbf{z}|\mathbf{x})$, the goal of the possibilistic nonlinear filter is to estimate recursively the posterior possibility function of the state, denoted $\pi_{k|k}(\mathbf{x}|\mathbf{z}_{1:k})$, where $\mathbf{z}_{1:k}$ is the sequence of target-originated measurements up to time k. Assuming the initial $\pi_0(\mathbf{x})$ at $k = 0$ is known, the solution can be presented in two stages: prediction and update [7], [8]. The prediction equation is given by

$$\pi_{k|k-1}(\mathbf{x} \mid \mathbf{z}_{1:k-1}) = \sup_{\mathbf{x}'\in\mathbb{X}} \rho_{k|k-1}(\mathbf{x} \mid \mathbf{x}')\pi_{k-1|k-1}(\mathbf{x}' \mid \mathbf{z}_{1:k-1}), \tag{1}$$

and it represents the possibilistic analogue of the Chapman-Kolmogorov equation. The update equation is given by

$$\pi_{k|k}(\mathbf{x} \mid \mathbf{z}_{1:k}) = \frac{g(\mathbf{z}_k \mid \mathbf{x})\pi_{k|k-1}(\mathbf{x} \mid \mathbf{z}_{1:k-1})}{\sup_{\mathbf{x}\in\mathbb{X}} g(\mathbf{z}_k \mid \mathbf{x})\pi_{k|k-1}(\mathbf{x} \mid \mathbf{z}_{1:k-1})}, \quad (2)$$

and it represents the possibilistic analogue of the Bayes update. The only difference between the (standard) probabilistic formulation of nonlinear filtering [9, Ch.1] and the possibilistic formulation expressed by (1) and (2) is that integrals in the former are replaced with supremums in the latter. The possibilistic nonlinear filter is a special instance of an outer measure class of nonlinear filters defined in [7].

Application of the possibilistic nonlinear filter to spatiotemporal tracking using natural language statements was studied in [3]. The filter was implemented using a grid-based method. Application to target motion analysis (TMA) using bearings-only measurements was presented in [8]. The filter was implemented using a particle filter. The conclusions of [8] are noteworthy: in the absence of a model mismatch, the probabilistic TMA and the possibilistic TMA filters perform identically. However, if there is a model mismatch, either in the dynamic model or in the measurement model, the possibilistic TMA filter is more robust, resulting in a significantly lower rate of filter divergences. Application to space object tracking was presented in [10].

AN OVERVIEW OF RECENT DEVELOPMENTS

The Bernoulli filter for single-target joint detection and tracking in the presence of false detections and misdetections was developed in the possibilistic framework for two cases: for a point target in [11] and for an extended target in [12]. In both cases, it was demonstrated that the possibilistic approach is more robust if the probability of detection is known only as an interval value.

The analogue of the probability hypothesis density (PHD) filter, for joint estimation of the number of targets and their states, was derived in the framework of possibility theory in [4]. This filter provides modelling flexibility in terms of facilitating the introduction of measurement-driven birth schemes and modelling the absence of information on the initial number of targets. However, it loses the ability of the standard PHD filter to estimate the number of targets by integration of the intensity function.

The first multitarget tracking algorithm in the probabilistic framework was reported in [13]. It was developed as a possibilistic analogue of the δ-generalised labelled multi-Bernoulli (δ-GLMB) filter. As such, it inherits all the capabilities of the standard probabilistic δ-GLMB filter, with the additional ability to deal with partial knowledge of dynamic model parameters, measurement model parameters, and the initial number and states of newborn targets. The possibilistic δ-GLMB filter is implemented using the concept of a Gaussian max-mixture (a weighted combination of Gaussian possibility functions).

A reward function for sensor control using the possibilistic nonlinear filter was studied in the context of bearings-only tracking in [14]. The reward was defined as the uncertainty reduction, where a measure of uncertainty contained in a posterior $\pi_{k|k}(\mathbf{x}|\mathbf{z}_{1:k})$ is defined as the volume under $\pi_{k|k}(\mathbf{x}|\mathbf{z}_{1:k})$.

SUMMARY

The formulation of target tracking algorithms in the framework of possibility theory is an exciting recent development. However, one should see it not as a 'silver bullet' for all situations but rather as an alternative to the standard Bayesian framework, with the potential to provide an additional layer of robustness due to epistemic uncertainty. Although early studies suggest promising results, further work is necessary to establish in a more universal context the benefits and pitfalls of the proposed framework.

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