DISTRIBUTED ESTIMATION

Chee-Yee Chong
Independent Researcher
Los Altos, CA, USA
cychong@ieee.org

Felix Govaers
Wolfgang Koch
Fraunhofer FKIE
Wachtberg, Germany
felix.govaers@fkie.fraunhofer.de
w.koch@ieee.org

Distributed estimation addresses the problem of combining local estimates that are based on measurements of individual sensors. This setup is particularly useful in spatially distributed applications, since data transmitted over potentially low-bandwidth communication links is reduced and the computational burden is shared among multiple nodes. Since local estimates have errors that are correlated over time for the same sensor, and across sensors at the same time, most distributed estimation research has focused on how to address this correlation [1], either by removing double counting to reconstruct the centralized estimate, or using the correlation to find the best linear estimate given the local estimates.

FUSION BY DECORRELATION

Early research on distributed estimation aims at reconstructing the centralized Kalman filter estimate $\hat{x}$ and error covariance $\hat{P}$ from local Kalman filter estimates $\hat{x}_i$ with covariances $\hat{P}_i$, *for i = 1,..,S.* The fusion equation [2] is:

$$\hat{P}^{-1}\hat{x}^{-1} = \bar{P}^{-1}\bar{x}^{-1} + \sum_{i}^{S}\left(\hat{P}_i^{-1}\hat{x}_i^{-1} - \bar{P}_i^{-1}\bar{x}_i^{-1}\right) \quad (1)$$

where $\bar{x}$, $\bar{P}$, $\bar{x}_i$, $\bar{P}_i$ are the global and local predictions. There is a similar equation for the error covariances.

Eq. (1) is sometimes called information matrix fusion because it is based on the information matrix form of the Kalman filter. It is also known as tracklet or equivalent measurement fusion [3] because the summand in (1) represents the new information in the measurements in the local estimates between fusion times. Because of its intuitive form and simple implementation, it is widely used in track fusion, and shown to have good performance even when the underlying assumptions of zero process noise or full-rate communication are violated.

Since a set of measurements is conditionally independent given the states at multiple observation times, optimal state estimate fusion can be achieved by using an augmented state that consists of the states at multiple times. The fusion equations have the same form as (1) except that x is the augmented state. Good estimation performance can be obtained using a small number of augmented states but the main benefit of using augmented state estimates is in track association [4].

FUSION OF PSEUDO ESTIMATES

Fusion of local estimates to compute the exact Kalman filter estimate in the presence of process noise and non-full-rate communication had been a challenging research problem for many years. This solution to this problem was published as the distributed Kalman filter (DKF) [5]:

$$\hat{P}^{-1}\hat{x}^{-1} = \sum_{i}^{S}\check{P}_i^{-1}\check{x}_i \quad (2)$$

where $\check{x}_i$ and error covariance $\check{P}_i$ are pseudo estimates different from the local Kalman estimates because either the prediction or update equations use global information. If the local knowledge deviates from the actual model, then the fusion equation will not produce the global estimate. The performance of DKF with pseudo estimates is compared with tracklet fusion in [6]. The distributed accumulated state density (DASD) filter [7] has a similar fusion equation but uses the ASD, which is the density of the augmented state.

The DKF and DASD filter can compute the optimal global estimate with no assumptions on the process noise and communication rate. However, the local estimates are pseudo-estimates and not Kalman estimates. Furthermore, the local pseudo-estimates are computed with global models; thus, these algorithms are more suitable for distributed processing and not for distributed estimation or fusion of local tracks.

ESTIMATE FUSION USING CROSS-COVARIANCE

Another popular fusion approach does exactly the opposite of decorrelation by exploiting the covariances and cross-covariances of the local estimates. This has advantages such as ignoring the dependence of the estimates due to prior communication and process noise, and the need to identify additional local estimates for decorrelation. However, the result is a constrained estimate which may be different from the centralized estimate given all the measurements.

The earliest work using this approach is the Bar-Shalom Campo rule [8]. Since the late 1990s, estimate fusion given the cross covariance has become a very active area of research because of its general applicability. Two popular ones are the maximum a posteriori (MAP) estimation [9], and the best linear unbiased estimation (BLUE) or weighted least-squares (WLS), [10], both first presented at FUSION 1999.

For two sensors, the MAP estimate has the same form as (1):

$$\hat{x} = \bar{x} + L_1(\hat{x}_1 - \bar{x}) + L_2(\hat{x}_2 - \bar{x}) \tag{3}$$

However, the gain matrix $L = [L_1, L_2]$ is calculated from the covariance matrix between the state x and the local estimates $[\hat{x}_1, \hat{x}_2]$. The BLUE or WLS [10] is a generalization of the MAP approach and can handle arbitrary correlations in the local estimates. For estimates $(\hat{x}_i)_{i=1}^{S}$ to be fused, a BLUE fusion rule is:

$$\hat{x} = \sum_{i=1}^{S} W_i \hat{x}_i \tag{4}$$

where the matrix weights $(W_i)_{i=1}^{S}$ are computed using the error covariances and satisfy the condition $\sum_{i=1}^{S} W_i = I$.

BLUE is very flexible because it can handle all types of local inputs and arbitrary correlations as long as the covariance between the inputs are known. The performance of BLUE fusion rules depends on the choice of the local inputs, and implementation requires knowing all the covariances. If the local inputs are chosen properly, the BLUE fusion rule can generate the centralized estimate given the measurements in the local estimates, such as augmented state estimate fusion. However, BLUE does not provide guidelines for selecting the local state estimates.

COVARIANCE INTERSECTION

The covariance intersection (CI) algorithm [11] was motivated by map building, where the cross-covariance between the thousands of variables is hard to model. CI assumes no knowledge of cross covariance. For two local estimates $\hat{x}_1$ and $\hat{x}_2$ with error covariances P_{11} and P_{22}, the CI algorithm is:

$$P_{CI}^{-1}\hat{x}_{CI} = \omega P_{11}^{-1}\hat{x}_1 + (1-\omega) P_{22}^{-1}\hat{x}_2 \tag{5}$$

where $\omega \in [0,1]$ is a parameter to be chosen such that the fused covariance P_{CI}^{-1} is minimal. CI produces a consistent estimate with a conservative error covariance. It is very popular when very little information on correlation in the estimates is available. The recently developed inverse covariance intersection [12] yields a good compromise between the conservative CI and other optimistic fusion rules such as naïve fusion. This is achieved by an application of the CI rule on the joint information of local estimates.

CONSENSUS FILTERS FOR SENSOR NETWORKS

Distributed estimation over a sensor network is difficult because the local estimates have correlations that depend on the information path. Fusion by decorrelation or using cross-invariances requires communication to share model and network information. Since communication is expensive, distributed estimation requires robust algorithms that assume only local network information, with performance measured by other metrics besides estimation accuracy. When distributed estimation is used to support distributed control, consensus in the estimates is more important than estimation accuracy. Thus, consensus filtering has become a very active area of research since the early 2000s [14]. It is based on the principle that a consensus estimate can be obtained by exchanging local information between observation times.

CONCLUSIONS

Much progress has been made in advancing the state of the art in distributed estimation over the past 25 years. However, not much has been done to provide guidance on selecting the appropriate algorithm for a particular problem. Further research to characterize the estimation performance, communication and computation requirements, and robustness of the algorithms is needed. Standard data sets and performance metrics will facilitate algorithm development and testing.

REFERENCES

1. Chong, C.-Y., Chang, K.-C., and Mori, S. Forty years of distributed estimation. In *Proc. 22nd Int. Conf. Information Fusion,* Ottawa, Canada, 2019.
2. Chong, C. Y. Hierarchical estimation. In *Proc. 2nd MIT/ONR Workshop Distributed Information and Decision Systems Motivated by Naval Command Control Communication Systems*, Monterey, CA, 1979.
3. Miller, M. D., Drummond, O. E., and Perrella, A. J. Tracklets and covariance truncation options for theater missile tracking. In *Proc. 1st Int. Conf. Multisource-Multisensor Data Fusion*, Las Vegas, NV, 1998.
4. Chong, C. Y., and Mori, S. Track association using augmented state estimates. In *Proc. 18th Int. Conf. Information Fusion*, Washington, DC, 2015.
5. Govaers, F., and Koch, W. Distributed Kalman filter fusion at arbitrary instants of time. In *Proc. 13th Int. Conf. Information Fusion*, Edinburgh, UK, 2010.
6. Chong, C. Y., Mori, S., Govaers, F., and Koch, W. Comparison of tracklet fusion and distributed Kalman filter for track fusion. In *Proc. 17th Int. Conf. Information Fusion*, Salamanca, Spain, 2014.
7. Koch, W., and Govaers, F. On decorrelated track-to-track fusion based on accumulated state densities. In *Proc. 17th Int. Conf. Information Fusion*, Salamanca, Spain, 2014.
8. Bar-Shalom, Y., and Campo, L. The effect of the common process noise on the two-sensor fused track covariance. *IEEE Trans. Aerospace and Electronic Systems*, Vol. 22, 6 (Nov. 1986), 803–805.
9. Mori, S., Barker, W. H., Chong, C. Y., and Chang, K. C. Track association and track fusion with non-deterministic target dynamics. In *Proc. 2nd Int. Conf. Information Fusion*, Sunnyvale, CA, 1999.
10. Zhu, Y., and Li, X. R. Best linear unbiased estimation fusion. In *Proc. 2nd Int. Conf. Information Fusion*, Sunnyvale, CA, 1999.
11. Uhlmann, J. K. General data fusion for estimates with unknown cross covariances. In *Proc. SPIE, AeroSense Conf*erence, Vol. 2755, 1996.
12. Noack, B., Sijs, J., Reinhardt, M., Hanebeck, U. D. Decentralized data fusion with inverse covariance intersection. *Automatica*, Vol. 79, (May 2017), 35–41.
13. Olfati-Saber, R., and Shamma, J. S. Consensus filters for sensor networks and distributed sensor fusion. In *Proc. 44th IEEE Conf. Decision Control*, Seville, Spain, 2005.