Bias Estimation for Collocated Sensors: Model Identification and Measurement Fusion

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The sensor bias estimation problem is crucial in autonomous driving systems for perception and target tracking. This work considers the bias estimation for two collocated synchronized sensors with slowly varying additive biases. The differences between the two sensors' observations are used to eliminate the target state. Consequently, the bias estimation is independent of the target-state estimation. The biases' observability condition is met when the two sensors' biases are Ornstein–Uhlenbeck stochastic processes with different time constants. The bias models, including the time constants and measurement noises, can be identified based on a sample autocorrelation or using the maximum-likelihood estimation technique. A maximum-likelihood measurement fusion technique is introduced for the bias-compensated observations. Simulation results, for several scenarios with various bias model parameters, prove the consistency of the estimator. It is shown that the uncertainties of biases are significantly reduced by the estimation algorithm presented. The sensitivity of the proposed algorithm is also tested with mismatched filters as well as the estimated bias models. Finally, the benefits of bias estimation in measurement fusion are evaluated.

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I. INTRODUCTION

Target tracking has always been an important problem for autonomous driving systems where multiple sensors are utilized to improve the tracking accuracy. Unfortunately, these sensors, such as radars, lidars, and cameras, are prone to biases, which can lead to a problematic association and hence poor results in target tracking. The sensors used in autonomous driving vehicles can only be placed together or very close (practically collocated), which makes the bias estimation challenging. Consequently, only a few works partially addressed this problem. Sensor calibration via off-line preprocessing is supposed to eliminate the sensor biases. However, it requires the knowledge of ground truth, and to be of value, the biases must be time-invariant. For the case where the sensor biases are dynamic and slowly varying, off-line calibration is not sufficient. This work proposes a real-time bias-estimation method for collocated sensors independent of the target-motion tracking and approaches for identifying the bias models.

Kastella and Yeary [7] considered the bias-estimation problem for radars on moving platforms by decoupling the tracking of targets of opportunity and the estimation of the radar and platform biases. Lin et al. [10] solved exact bias estimation for an active sensor by using pairs of range and angle measurements to create pseudo-measurements of the biases of both sensors relying on the nonlinearity of the range and angle measurements. Bar-Shalom [3] considered time-varying bias estimation along with the target state. This work is based on [3] and uses the subtraction between the sensor measurements as in [10] to eliminate the target state in estimating the sensor biases, i.e., the biases can be estimated independently of the target. In [4], the authors considered the problem of estimating sensor biases from measurements of targets flying on known trajectories by augmenting the kinematic state vector with sensor bias parameters. In [10] and [11], the bias model includes scale biases and unknown locations of the sensors, as well as the usual offset biases. Kowalski et al. [8] considered three-dimensional sensor bias estimation using sine space measurements and showed the achievability of the Cramér–Rao lower bound.

To handle bias estimation for collocated sensors, this work considers slowly varying sensor biases that are modeled as Ornstein–Uhlenbeck (OU, a class of Gauss–Markov) processes (as discussed in [3]) and deals with bias estimation with the following contributions: (i) solving the problem for collocated synchronized sensors; (ii) estimation of biases independent of target motion by using the difference between the associated sensor observations; and (iii) application of the proposed method to all kinds of observations (i.e., bearings, range, etc.) from various types of sensors.

The bias models (parameters of the OU process) are typically unknown with limited prior information. In [12] and [13], the mean-reverting OU process parameter estimation is discussed along with a long-term prediction. Two approaches are introduced in this paper to identify the bias model: (i) sample autocorrelation based method; and (ii) maximum-likelihood (ML) estimation of the model parameters. The prior information about the sensor biases consists of initial distributions, assumed to be Gaussian with zero mean and certain variances. After the bias estimation, one can fuse the local observations, according to Fusion Configuration III [2], with the bias compensation taking into account the error in the bias estimates. The fusion is carried out using the ML criterion. The performance of the proposed method is tested via simulations based on Monte Carlo (MC) runs by showing the reduction in the mean-square (MS) error of the bias-compensated fused measurements versus fusion without compensation. The estimator and fuser presented are shown to be consistent.

The process flowchart for bias estimation, fusion, and system identification is shown in Fig. 1. This paper is organized as follows. Section II formulates the problem by introducing the bias dynamic model and the bias measurement model (using the subtraction between the sensor observations) and discusses the bias observability. The bias model identification methods are presented in Section III. In Section IV, the fusion of the bias-compensated observations is presented. Section V gives the simulation results of several scenarios. Conclusions and remarks are in Section VI.

II. PROBLEM FORMULATION

The challenge of this work is to estimate the (collocated) sensor biases efficiently given synchronized observations defined as

$$z_1(k) = h\left[\mathbf{x}(k), \mathbf{s}(k)\right] + b_1(k) + w_1(k), \quad k = 1, 2, \ldots, N, \tag{1}$$

and

$$z_2(k) = h\left[\mathbf{x}(k), \mathbf{s}(k)\right] + b_2(k) + w_2(k), \quad k = 1, 2, \ldots, N, \tag{2}$$

where $\mathbf{x}$ is the true (common) target state, which is unknown, $\mathbf{s}$ is the sensor state, and $h[\cdot, \cdot]$ is the generic observation model (angle or range). Since the sensors are collocated, they share the same sensor motion. The observations obtained from the sensors depend on both the sensor and target motions as well as the biases and noises. The bias estimation introduced in the following does not require the target state. The observation noises

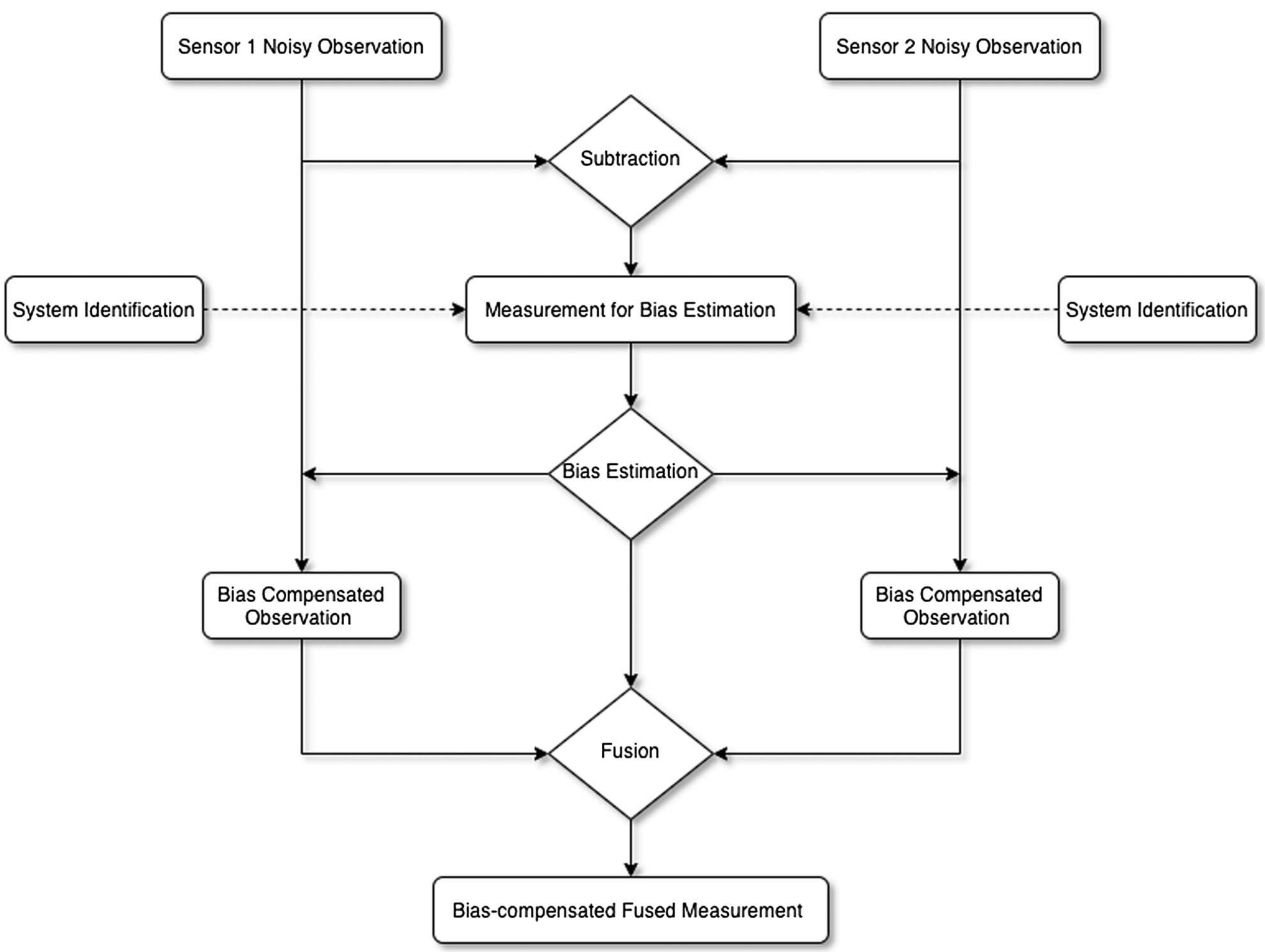


Fig. 1. Process flowchart for bias estimation and fusion.

w_1 and w_2 are assumed to be zero mean, white with variances $\sigma_{w_1}^2$ and $\sigma_{w_2}^2$, and are independent of each other and of the sensor biases. The above observations are not restricted to the type of sensors; only the same kind of observation is required, i.e., radars, lidars, and cameras can provide angle observations; lidars and radars can also provide range observations. It will be shown in the sequel that bias estimation is independent of the target-state estimation, i.e., bias estimation and state tracking are decoupled. The observations considered are using generic models in one coordinate (i.e., with a dimension of 1) to aid in the clarity of the exposition.

The sensor biases are slowly varying, modeled as an OU (first-order Gauss–Markov) process [7]. The discrete dynamic model used for the biases [1], [3] is (two collocated sensors are considered in this work)

$$b_i(k+1) = \alpha_i b_i(k) + v_i(k), \qquad i = 1, 2, \tag{3}$$

with

$$\alpha_i = e^{-T/\tau_i}, \tag{4}$$

where T is the sampling interval and τ_i is the time constant of the bias evolution (assumed to be known; its estimation is discussed in Section III).

The time constant is given in terms of α_i as

$$\tau_i = -T \ln \alpha_i. \tag{5}$$

The above expression can be rewritten using the first-order Taylor expansion for $\tau \gg T$ as

$$\tau_i \approx \frac{T}{1-\alpha_i}. \tag{6}$$

The driving process noises v_i are assumed to be zero mean, white with variances $\sigma_{v_i}^2$. All the noises v_i and w_i are independent. Using the above model guarantees that the bias estimates are bounded since (3) is stable.

The MS value of the bias b_i is $\sigma_{b_i}^2$ and its relationship to the corresponding process noise variance is

$$\sigma_{v_i}^2 = (1-\alpha_i^2)\sigma_{b_i}^2. \tag{7}$$

Since the sensor biases b_i will be estimated independently of the target state, only the difference between the observations is used:

$$z(k) = z_1(k) - z_2(k) = b_1(k) - b_2(k) + w_1(k) - w_2(k). \tag{8}$$

The bias state to be estimated is

$$\mathbf{b}(k) = [b_1(k)\ b_2(k)]' \tag{9}$$

with the state equation

$$\mathbf{b}(k+1) = F\mathbf{b}(k) + \mathbf{v}(k), \tag{10}$$

where

$$F = \begin{bmatrix} \alpha_1 & 0 \\ 0 & \alpha_2 \end{bmatrix}, \tag{11}$$

and the process noise vector is

$$\mathbf{v} = [v_1\ v_2]' \tag{12}$$

with the covariance matrix

$$Q = \begin{bmatrix} \sigma_{v_1}^2 & 0 \\ 0 & \sigma_{v_2}^2 \end{bmatrix}. \tag{13}$$

The measurement model based on (8) is

$$z(k) = H\mathbf{b}(k) + w(k), \tag{14}$$

where

$$H = \begin{bmatrix} 1 & -1 \end{bmatrix} \tag{15}$$

and the measurement noise $w(k)$ is

$$w(k) = w_1(k) - w_2(k), \tag{16}$$

which has variance $\sigma_{w_1}^2 + \sigma_{w_2}^2$.

A Kalman filter (KF) is then used for the estimation of $\mathbf{b}(k)$, which gives the estimate at time k [1]

$$\hat{\mathbf{b}}(k) = [\hat{b}_1(k)\ \hat{b}_2(k)]'. \tag{17}$$

The observability of the above system can be verified via the observability matrix [9]

$$\mathcal{O} = \begin{bmatrix} H \\ HF \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ \alpha_1 & -\alpha_2 \end{bmatrix}, \tag{18}$$

which has full rank under the conditions (i) $\alpha_1 \neq \alpha_2$, and (ii) none is unity.[1] Assuming that the sensors are different, the bias models will be different due to their physical properties, i.e., different α_i in (3). Using a discrete-time Wiener process to model both sensor biases will lead to lack of observability of the system with $\alpha_i = 1$, and the bias becomes the integral (sum) of the white noise sequence terms (and diverges). Thus, both the observability conditions can be met with reasonably realistic assumptions.

It can be easily seen that the pair (F, C), where C is the Cholesky factor of the covariance matrix Q, is completely controllable. Thus, the solution of the discrete-time Riccati equation for such a time-invariant system will converge to a finite steady-state (SS) covariance [1], which can obtained via KF.

III. BIAS MODEL IDENTIFICATION

The sensor biases are assumed to be OU processes, as shown in (3); however, the time constants τ_i are, in general, unknown. To identify the system (bias model) as well as the process and measurement noise variances, two approaches are introduced: (i) system parameter estimation based on sample autocorrelation, and (ii) ML estimation. Note that the bias model identification is independent of and prior to the bias estimation.

For simplicity, only one system is analyzed for illustration; however, the approach can be used for multiple sensors with the same model but different parameters.

[1] The bias model should not diverge.

Consider the discrete-time OU bias model (with a slowly varying bias[2])

$$b(k+1) = \alpha b(k) + v(k) \tag{19}$$

and noisy observation model ($k = 1, 2, \ldots, N$)

$$z = h\left[\mathbf{x}(k), \mathbf{s}(k)\right] + b(k) + w(k), \tag{20}$$

$$o(k) \triangleq z(k) - h\left[\mathbf{x}(k), \mathbf{s}(k)\right] = b(k) + w(k), \tag{21}$$

with α defined in (4), v is the process noise with variance σ_v^2, and w is the measurement noise with variance σ_w^2; $\mathbf{x}$ and $\mathbf{s}$ are as defined in (2). The observation $o(k)$ is obtained assuming that the truth is known, which can be done in the off-line precalibration. However, note that $h\left[\mathbf{x}(k), \mathbf{s}(k)\right]$ will cancel in (8), which will be used in estimating the two biases (17). The identification of the bias models will rely on (21).

A. Bias Model Parameter Estimation Using Sample Autocorrelation

The autocorrelation of $o(k)$, assumed to be wide-sense stationary (WSS), is

$$\begin{aligned} r(m) &= \mathrm{E}\{o(k)o(k-m)\} \\ &= \mathrm{E}\{[b(k)+w(k)][b(k-m)+w(k-m)]\} \\ &= \mathrm{E}\{b(k)b(k-m)\} + \sigma_w^2\delta(m) \\ &= r_b(m) + \sigma_w^2\delta(m), \end{aligned} \tag{22}$$

where $\delta(k)$ is the Kronecker delta function and $r_b(m)$ is the autocorrelation of $b(k)$, also assumed to be WSS,

$$\begin{aligned} r_b(m) &= \mathrm{E}\{b(k)b(k-m)\} \\ &= \mathrm{E}\{[\alpha b(k-1) + v(k-1)][b(k-m)]\} \\ &= \alpha^m r_b(0) \end{aligned} \tag{23}$$

and

$$\begin{aligned} r_b(0) &= \mathrm{E}\{b(k)b(k)\} \\ &= \mathrm{E}\{[\alpha b(k-1) + v(k-1)] \\ &\quad \cdot [\alpha b(k-1) + v(k-1)]^*\} \\ &= \alpha^2 r_b(0) + \sigma_v^2. \end{aligned} \tag{24}$$

The above equation yields

$$r_b(0) = \frac{\sigma_v^2}{1-\alpha^2}. \tag{25}$$

Substituting (25) into (22) gives

$$r(m) = \alpha^m \frac{\sigma_v^2}{1-\alpha^2} + \sigma_w^2\delta(m), \tag{26}$$

which can be used to estimate α, σ_v^2, and σ_w^2. That is, assuming that the sample autocorrelations are available (sufficiently accurate since one can have only sample autocorrelations, i.e., time averages), one has

$$\alpha = \frac{r(2)}{r(1)}, \tag{27}$$

$$\sigma_v^2 = \frac{r(1)^2 - r(2)^2}{r(2)}, \tag{28}$$

$$\sigma_w^2 = r(0) - \frac{r(1)^2}{r(2)}. \tag{29}$$

Note that the above solution is highly dependent on the accuracy of sample autocorrelations (especially when α is very close to 1), which cannot be guaranteed with limited sample data. Assuming that more autocorrelations (i.e., more than $r(2)$) are available, the parameters can be estimated using all sample autocorrelations to improve the accuracy as discussed in the following.

Equation (26) can be written for $m > 0$ as

$$\ln r(m) = m\beta + \gamma, \qquad m > 0, \tag{30}$$

where

$$\beta = \ln \alpha, \tag{31}$$

$$\gamma = \ln \frac{\sigma_v^2}{1-\alpha^2}. \tag{32}$$

The least-squares (LS) estimate of parameters $\hat{\beta}$ and $\hat{\gamma}$, given $(r(1), \ldots, r(M))$, can be obtained as (see Appendix)

$$\begin{bmatrix} \hat{\beta} \\ \hat{\gamma} \end{bmatrix} = \begin{bmatrix} \sum_{m=1}^M m^2 & \sum_{m=1}^M m \\ \sum_{m=1}^M m & \sum_{m=1}^M 1 \end{bmatrix}^{-1} \begin{bmatrix} \sum_{m=1}^M m \ln r(m) \\ \sum_{m=1}^M \ln r(m) \end{bmatrix}, \tag{33}$$

and, using (31) and (32), one has

$$\hat{\alpha} = e^{\hat{\beta}}, \tag{34}$$

$$\hat{\sigma}_v^2 = (1 - e^{2\hat{\beta}})e^{\hat{\gamma}}. \tag{35}$$

The full set of model parameter estimates is given by (34), (35), and (29). Note that the parameter σ_w^2 does not require an additional LS estimator since it is not sensitive to the accuracy of sample autocorrelations since $\sigma_v^2 \ll \sigma_w^2$.

B. ML Estimation of the System Parameters

The above equations can provide an explicit solution for system parameter estimation in terms of the sample autocorrelations, and due to computational and speed demands, this is of high interest. However, the accuracy of these estimated autocorrelations is highly dependent on the data batch length. Alternatively, the ML approach can be used for estimating the system parameters of interest [5], [6].

[2]This implies that α is near unity and σ_v is small.

The observation model (21) can be modified taking into account the bias model (19) as

$$o(k) = \alpha o(k-1) + w(k) - \alpha w(k-1) + v(k-1)$$
$$= \alpha o(k-1) + u(k), \tag{36}$$

where

$$u(k) \triangleq w(k) - \alpha w(k-1) + v(k-1) \tag{37}$$

is zero mean but not white.

Consider an observation batch of length $L\ (L \leq k-1)$

$$\mathbf{O}^L(k) = \begin{bmatrix} o(k) \\ o(k-1) \\ \cdots \\ o(k-L+1) \end{bmatrix}$$
$$= \alpha \begin{bmatrix} o(k-1) \\ o(k-2) \\ \cdots \\ o(k-L) \end{bmatrix} + \begin{bmatrix} u(k) \\ u(k-1) \\ \cdots \\ u(k-L+1) \end{bmatrix}$$
$$= \alpha \mathbf{O}^L(k-1) + \mathbf{U}^L(k). \tag{38}$$

The noise vector $\mathbf{U}^L(k)$ is zero mean with a covariance matrix of dimension L

$$R^U(L) \approx \begin{bmatrix} 2\sigma_w^2 & -\alpha\sigma_w^2 & & 0 \\ -\alpha\sigma_w^2 & \ddots & \ddots & \\ & \ddots & \ddots & -\alpha\sigma_w^2 \\ 0 & & -\alpha\sigma_w^2 & 2\sigma_w^2 \end{bmatrix} \tag{39}$$

under the assumptions that (i) α is very close to 1, and (ii) the effect of the process noise v is negligible [$\sigma_v^2 \ll \sigma_w^2$, which can be seen from (7) as the difference on the right-hand side is close to zero] and will not be estimated.[3]

The likelihood function (LF) of α and σ_w^2 based on (38) is

$$\Lambda(\alpha, \sigma_w^2 | \mathbf{O}^L(k))$$
$$= |2\pi R^U(L)|^{-1/2} \exp\left[\frac{1}{2}\Delta\mathbf{o}(k)'[R^U(L)]^{-1}\Delta\mathbf{o}(k)\right], \tag{40}$$

where

$$\Delta\mathbf{o}(k) = \mathbf{O}^L(k) - \alpha\mathbf{O}^L(k-1). \tag{41}$$

The ML estimates (MLEs), $\hat{\alpha}$, and $\hat{\sigma}_w^2$ can be found by maximizing (40) via a numerical search. In practice, a two-dimensional grid can be used for this. The process noise σ_v^2 estimate can be obtained via (7) using

$$\hat{\sigma}_v^2 = \sigma_o^2 - \hat{\sigma}_w^2, \tag{42}$$

where σ_o^2 is the MS value of the noisy observation (21).

[3]The process noise variance σ_v^2 would appear added to each diagonal term.

IV. FUSION OF THE OBSERVATION WITH BIAS COMPENSATION

Under the Gaussian assumption, the fusion of bias-compensated observations can be solved using the ML criterion, i.e., by maximizing the LF or by minimizing the negative log-likelihood function (NLLF) of the target position based on the observations from the two sensors.[4] Note that for the linear Gaussian case, the LS estimator and ML estimator, as well as the minimum MS error (MMSE) estimator, coincide [1], [2].

With the bias estimates, the current observation can be expressed as

$$z_i(k) = \zeta(k) + \hat{b}_i(k) + \tilde{b}_i(k) + w_i(k), \qquad i = 1, 2, \tag{43}$$

where

$$\zeta(k) = h[\mathbf{x}(k), \mathbf{s}(k)] \tag{44}$$

is the noiseless fused observation that needs to be estimated given $z_i(k)$ and the bias estimates $\hat{b}_i(k)$, and by accounting for the residual bias error $\tilde{b}_i(k)$. The estimates for sensor biases are obtained using a KF and then we directly estimate the fused observation with bias compensation.

The bias-compensated ("bc") observations, omitting the time argument k for simplicity, are

$$z_1^{\text{bc}} = z_1 - \hat{b}_1 = \zeta + \tilde{b}_1 + w_1 \tag{45}$$

and

$$z_2^{\text{bc}} = z_2 - \hat{b}_2 = \zeta + \tilde{b}_2 + w_2. \tag{46}$$

Under the ML criterion, the fusion is carried out by estimating ζ based on the bias-compensated observation vector $[z_1^{\text{bc}}\ z_2^{\text{bc}}]'$. The fused observation[5] with bias compensation ("Fbc") is [1, Eq. (3.4.1-9)]

$$\hat{\zeta}^{\text{Fbc}} = \left[(H^{\text{Fbc}})'(R^{\text{Fbc}})^{-1}H^{\text{Fbc}}\right]^{-1}$$
$$\cdot (H^{\text{Fbc}})'(R^{\text{Fbc}})^{-1}[z_1^{\text{bc}}\ z_2^{\text{bc}}]', \tag{47}$$

where, in this case,

$$H^{\text{Fbc}} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tag{48}$$

and

$$R^{\text{Fbc}} = E\left\{\begin{bmatrix} \tilde{b}_1 + w_1 \\ \tilde{b}_2 + w_2 \end{bmatrix}\begin{bmatrix} \tilde{b}_1 + w_1 & \tilde{b}_2 + w_2 \end{bmatrix}\right\} \tag{49}$$

$$= \begin{bmatrix} P_{11} + \sigma_{w_1}^2 & P_{12} \\ P_{12} & P_{22} + \sigma_{w_2}^2 \end{bmatrix}. \tag{50}$$

[4]The LF of a parameter of interest (in this case, the target position) is the probability density function (pdf) of the observation conditioned on the parameter [1]. In the literature, one can find the term "likelihood of the observation," which is incorrect.

[5]The ML estimator is implemented using the LS technique [1].

Table I
Steady-State Bias Error Variances and Standard Deviations for Each Scenario

Scenario	$(1-\alpha_1, 1-\alpha_2)$	(τ_1, τ_2)	P_{11}	$\sigma_{\hat{b}_1}$	P_{22}	$\sigma_{\hat{b}_2}$
1	$(10^{-4}, 10^{-2})$	$(10^3, 10^1)$	0.1673	0.4091	0.3084	0.5553
2	$(10^{-5}, 10^{-2})$	$(10^4, 10^1)$	0.0598	0.2445	0.2220	0.4712
3	$(10^{-4}, 10^{-3})$	$(10^3, 10^2)$	0.3689	0.6074	0.4014	0.6336

Table III
Bias NEES for Each Scenario With Various Numbers of Sampling Scans From 100 Runs; 95% Probability Interval is [1.63 2.41]

Scenario	$N = 500$	$N = 1000$	$N = 2000$
1	1.7321	2.1496	2.1339
2	2.1413	2.0102	1.8412
3	2.0802	2.1724	2.0791

In (50), P_{mn} is the (m, n) element of the calculated covariance matrix associated with the bias estimate vector (17).

The variance corresponding to the fused bias-compensated observation (47) is

$$P^{\text{Fbc}} = \left[(H^{\text{Fbc}})'(R^{\text{Fbc}})^{-1}H^{\text{Fbc}}\right]^{-1}. \quad (51)$$

The naïve fusion (with no bias compensation—"Fnbc") is

$$\hat{\zeta}^{\text{Fnbc}} = \frac{\sigma_{w_1}^{-2} z_1 + \sigma_{w_2}^{-2} z_2}{\sigma_{w_1}^{-2} + \sigma_{w_2}^{-2}}, \quad (52)$$

which has an MS error

$$P^{\text{Fnbc}} = \frac{\sigma_{w_1}^{-4}(\sigma_{w_1}^2 + \sigma_{b_1}^2) + \sigma_{w_2}^{-4}(\sigma_{w_2}^2 + \sigma_{b_2}^2)}{(\sigma_{w_1}^{-2} + \sigma_{w_2}^{-2})^2}. \quad (53)$$

Note that the (non-Bayesian) ML fusion technique is the same as the (Bayesian) MMSE fusion technique for dependent tracks (with Gaussian errors), as discussed in [2].

V. SIMULATION RESULTS

Numerical examples are shown in this section with simulation results. For simplicity and illustration, consider the stochastic biases to be estimated have MS values $\sigma_{b_1}^2 = \sigma_{b_2}^2 = 1$ for both sensors. The observation noises $\sigma_{w_1}^2$ and $\sigma_{w_2}^2$ share the same variance 1. The sampling interval $T = 0.1$ s. Simulation results are obtained based on 100 MC runs.

Table II
Calculated Bias Estimate Variances for Various Numbers of Scans for Each Scenario

Scans	Scenario	$(1-\alpha_1, 1-\alpha_2)$	(τ_1, τ_2)	P_{11}	P_{22}	P^{Fbc}
500	1	$(10^{-4}, 10^{-2})$	$(10^3, 10^1)$	0.2529	0.3786	0.7698
	2	$(10^{-5}, 10^{-2})$	$(10^4, 10^1)$	0.2308	0.3620	0.7505
	3	$(10^{-4}, 10^{-3})$	$(10^3, 10^2)$	0.4686	0.4959	0.9662
1000	1	$(10^{-4}, 10^{-2})$	$(10^3, 10^1)$	0.1952	0.3313	0.7171
	2	$(10^{-5}, 10^{-2})$	$(10^4, 10^1)$	0.1512	0.2969	0.6779
	3	$(10^{-4}, 10^{-3})$	$(10^3, 10^2)$	0.4405	0.4692	0.9388
2000	1	$(10^{-4}, 10^{-2})$	$(10^3, 10^1)$	0.1709	0.3113	0.6949
	2	$(10^{-5}, 10^{-2})$	$(10^4, 10^1)$	0.0963	0.2519	0.6277
	3	$(10^{-4}, 10^{-3})$	$(10^3, 10^2)$	0.4062	0.4363	0.9054

A. Numerical Results

Three scenarios are considered in this section with different values of the pair $(1-\alpha_1, 1-\alpha_2)$: $(10^{-4}, 10^{-2})$, $(10^{-5}, 10^{-2})$, and $(10^{-4}, 10^{-3})$. The corresponding time-constant pairs (τ_1, τ_2) are $(10^3, 10^1)$ s, $(10^4, 10^1)$ s, and $(10^3, 10^2)$ s. With the same sampling interval, a smaller $1 - \alpha_i$ (α_i closer to 1) indicates a larger time constant and the corresponding bias has a smoother trajectory [7]. As discussed at the end of Section II, the covariance of the bias estimates will converge to an SS value. Table I lists the solutions of the SS variances from the discrete-time Ricatti equation for each scenario. Note that for such a system with α_i close to 1, the convergence rate is slow due to the small Kalman gain. The calculated covariances (variances for each bias, specifically) for each scenario with respect to various numbers of scans N are shown in Table II. The total simulation duration is NT. The filter consistency is tested using the normalized estimation error squared (NEES) [1], which is Chi-square distributed with the number of degrees of freedom given by the number of MC runs (n_{MC}) and the dimension of the parameter vector (2 in this case). The 95% probability interval with $n_{\text{MC}} = 100$ for the bias estimate is [1.63 2.41]. It can be seen from Table III that the NEES[6] for all the cases falls into the above interval and the estimator is thus consistent, i.e., the actual MSE matches the filter-calculated variance.

In all the cases considered, the estimator reduced the uncertainty of the biases. In Scenario 2 with 1000 scans, the MS value of each bias is 1 before the biases are estimated, which is reduced to 0.1512 for b_1 (61% standard deviation reduction) and 0.2969 for b_2 (46% standard deviation reduction). Similarly, for Scenario 1 with 1000 scans, the standard deviation reduction is 56% for b_1 and 42% for b_2. It can be seen that with 2000 scans (total observation time 200 s), the calculated variance almost reaches its SS value. The performance of the estimator is sensitive to the bias models, i.e., when α_1 and α_2 are close, such as in Scenario 3, the estimation performance (uncertainty reduction) is not as significant due to the marginal observability (the observability matrix $\mathcal{O}$ being nearly singular).

In Table II, the variances of the fused observations (51) for each scenario are also shown. The uncompen-

[6]The number of degrees of freedom is $100 \times 2 = 200$ and the NEES uses division by 100, i.e., it should be around 2.

Table IV
Sensitivity of MSE of Each Bias Estimate ($\text{MSE}_{b_1}, \text{MSE}_{b_2}$) for Various Filter Model Time Constants ($\tau_1^{\text{MOD}}, \tau_2^{\text{MOD}}$) From 1000 Runs With True Model $\tau_1^{\text{TRUE}} = 10^3$ s, $\tau_2^{\text{TRUE}} = 10$ s. Note that Unless $\tau^{\text{MOD}} = \tau^{\text{TRUE}}$ the Filters Are Mismatched

		τ_2^{MOD}			
($\text{MSE}_{b_1}, \text{MSE}_{b_2}$)		$0.5\tau_2^{\text{TRUE}}$	τ_2^{TRUE}	$2\tau_2^{\text{TRUE}}$	$5\tau_2^{\text{TRUE}}$
τ_1^{MOD}	$0.5\tau_1^{\text{TRUE}}$	(0.1971, 0.3391)	(0.1826, 0.3163)	(0.1897, 0.3333)	(0.2358, 0.4155)
	τ_1^{TRUE}	(0.1812, 0.3236)	(0.1760, 0.3109)	(0.1846, 0.3309)	(0.2239, 0.4087)
	$2\tau_1^{\text{TRUE}}$	(0.1792, 0.3194)	(0.1801, 0.3145)	(0.1882, 0.3354)	(0.2228, 0.4102)
	$5\tau_1^{\text{TRUE}}$	(0.1884, 0.3240)	(0.1897, 0.3225)	(0.1947, 0.3423)	(0.2252, 0.4141)

Table V
Sensitivity of the Fused Observation MSE (MSE_{Fbc}) With Bias Compensation From 1000 Runs

		τ_2^{MOD}			
MSE_{Fbc}		$0.5\tau_2^{\text{TRUE}}$	τ_2^{TRUE}	$2\tau_2^{\text{TRUE}}$	$5\tau_2^{\text{TRUE}}$
τ_1^{MOD}	$0.5\tau_1^{\text{TRUE}}$	0.7041	0.6878	0.6991	0.7557
	τ_1^{TRUE}	0.6939	0.6863	0.6985	0.7479
	$2\tau_1^{\text{TRUE}}$	0.6956	0.6938	0.7050	0.7492
	$5\tau_1^{\text{TRUE}}$	0.7070	0.7056	0.7136	0.7533

sated sensor observation bias has MS values $\sigma_{b_i}^2 = \sigma_{w_i}^2 = 1$ and the naïvely fused error has an MS value $P^{\text{Fnbc}} = 1$ in all cases. After bias estimation and the compensated fusion, the MS values have been significantly reduced. The MS reduction is up to 37% (with $P^{\text{Fbc}} = 0.63$ for Scenario 2).

B. Sensitivity

In the real world, the true bias models are not available in most cases, which will result in a mismatched filter in estimation. The sensitivity of the proposed estimation method is shown in Table IV, where the MSEs of the bias estimate (obtained through a mismatched filter for Scenario 1) are listed. The true bias dynamic models have time constants $\tau_1^{\text{TRUE}} = 10^3$ s and $\tau_2^{\text{TRUE}} = 10^1$ s. To test the sensitivity of the proposed estimator, different time constants are considered in the KF. The biases' time constants used in the KF are τ_1^{MOD} and τ_2^{MOD}, respectively. The calculated variances for the bias estimates are independent of the actual measurements with $P_{11} = 0.1709$ and $P_{22} = 0.3113$ for all the cases considered. Since the bias dynamic models used in the filter are different from the true ones, the consistency is lost. It can be seen that, if the true bias model has time constants different from those assumed in the estimator, the MSE of the estimate increases. Nevertheless, there is always a reduction in the bias error uncertainty.

The corresponding MSEs of the fused observations[7] with bias compensation for various filter bias models are shown in Table V. The simulation results are obtained from 1000 MC runs. It can be seen that, even with mismatched models in the filter, the fusion of the observations with bias compensation always achieves a smaller MSE than the MSE of the "naïve" fusion, which is 1.

[7]These are MS errors, not covariances, in the case of mismatched filters.

Table VI
Sample Autocorrelation Based Estimation of Bias b_1 Model for Various Data Batch Sizes

Batch length	5×10^5	10^6	10^7
e_1	0.122	0.371	0.621
m_1	1.001	1.000	1.000

C. Bias Model Identification

In this subsection, the scenario with decorrelation true time constants $\tau_1^{\text{TRUE}} = 10^3$ s and $\tau_2^{\text{TRUE}} = 10^1$ s is considered. The two approaches introduced in Section III are tested in the following.

Define the ratio of the estimated time constant to the true constant as

$$e_i = \hat{\tau}_i / \tau_i^{\text{TRUE}}, \qquad i = 1, 2, \tag{54}$$

which indicates the accuracy of the estimation and can be used for sensitivity analysis, and

$$m_i = \hat{\sigma}_{w_i}^2 / (\hat{\sigma}_{w_i}^{\text{TRUE}})^2, \qquad i = 1, 2, \tag{55}$$

to be the ratio of the estimated measurement noise variance and the truth, taken as $(\hat{\sigma}_{w_i}^{\text{TRUE}})^2 = 1$. The simulation results of e_i and m_i from the sample autocorrelation based estimation of the corresponding bias model are shown in Tables VI and VII with various data batch lengths for $i = 1$ and $i = 2$, respectively. It can be seen that, with more data, the accuracy of the model estimation has been better. Note that bias model 1, with a higher time constant, requires a longer batch.

Table VII
Sample Autocorrelation Based Estimation of Bias b_2 Model for Various Data Batch Sizes

Batch length	5×10^4	10^5	5×10^5
e_2	0.650	0.736	0.916
m_2	0.995	0.986	0.999

Table VIII
MLE From 100 MC Runs (Batch Size $L = 250$)

i	$\Delta\alpha$	$\Delta\sigma_w^2$	$1-\alpha$ grid	σ_w^2 grid	$\bar{e}_i$	RMSE(e_i)	$\bar{m}_i$	RMSE(m_i)
1	0.001	0.01	[0.95 0.999]	[0.5 1.5]	5.489	4.611	1.000	0.087
2	0.00001	0.01	[0.999 0.99999]	[0.5 1.5]	1.369	1.265	1.001	0.095

The MLE is obtained via numerical search, where the search intervals for time constants and measurement noises as well as the grid steps are shown in Table VIII. The observation batch size is $L = 250$ and the simulation results [mean and variance of the ratios, (54) and (55), respectively] are based on 100 MC runs. The MSEs of bias estimation and fusion with bias compensation based on the result of system identification are are shown in Table IX for the two approaches discussed. The degradation of $\text{MSE}_{\text{FbcMI}}$ (MSE of fusion with bias compensation based on model identification) versus MSE_{Fbc} (MSE from correct filter) is around 11% for the autocorrelation-based model identification, and is around 6% for ML-based identification due to the estimated model in the filter. Note that the MSE of fused observation is the most important, and, even with a relatively high error in the bias model estimation, the fused MSE is clearly better with bias compensation than without. Also note that the ML procedure requires a much shorter batch length.

VI. CONCLUSIONS

In this work, the bias estimation for collocated synchronized sensors is solved using a target of opportunity. The sensor biases are slow varying, modeled as OU processes. Only the difference between the sensor observations is used for the bias estimation, which is thus independent of the target-state estimation. The system is observable when the biases have different eigenvalues in their noise-driven discrete time dynamic models. The bias model parameters can be obtained directly via sample autocorrelations or via ML estimation. With bias estimates and by accounting for the residual biases, the fusion of the bias-compensated observations is carried out under the ML criterion. The standard deviation of the fused measurement is significantly reduced. The performances, in terms of both accuracy and convergence time, are sensitive to and depend on the bias dynamics. The bias-estimation consistency is proved via simulation results. Using the ML criterion, the fusion of the observations carried out with bias compensation results in a significant MSE reduction for the fused measurement. The proposed algorithm is also shown to provide benefits even with mismatched filers with bias dynamic models different from the true ones.

Table IX
MSE of Fused Observation Based on System Identification From 100 Runs

Bias estimation approach	MSE_{b_1}	MSE_{b_2}	$\text{MES}_{\text{FbcMI}}$
Autocorrelation based (batch length 5×10^6)	0.2278	0.3416	0.7728
ML (batch length 250)	0.1650	0.3351	0.7378

APPENDIX

A LEAST SQUARES ESTIMATOR OF BIAS MODEL PARAMETERS

Given the sample autocorrelations $\{r(1), r(2), \ldots, r(M)\}$, the LS estimates are

$$\begin{bmatrix}\hat{\beta}\\ \hat{\gamma}\end{bmatrix} = \arg\min_{\beta,\gamma} E(\beta,\gamma)$$

$$= \arg\min_{\beta,\gamma}\left\{\sum_{m=1}^{M}[\ln r(m) - (m\beta+\gamma)]^2\right\}. \quad \text{(A1)}$$

Minimization of the above expression

$$\frac{\partial E(\beta,\gamma)}{\partial\beta} = 0, \frac{\partial E(\beta,\gamma)}{\partial\gamma} = 0, \quad \text{(A2)}$$

where

$$\frac{\partial E(\beta,\gamma)}{\partial\beta} = \sum_{m=1}^{M} -2[\ln r(m) - (m\beta+\gamma)]m, \quad \text{(A3)}$$

$$\frac{\partial E(\beta,\gamma)}{\partial\gamma} = \sum_{m=1}^{M} -2[\ln r(m) - (m\beta+\gamma)]. \quad \text{(A4)}$$

Setting $\frac{\partial E(\beta,\gamma)}{\partial\beta} = \frac{\partial E(\beta,\gamma)}{\partial\gamma} = 0$ gives

$$\sum_{m=1}^{M}[\ln r(m) - (m\beta+\gamma)]m = 0, \quad \text{(A5)}$$

$$\sum_{m=1}^{M}[\ln r(m) - (m\beta+\gamma)] = 0. \quad \text{(A6)}$$

Rewriting the above equations as

$$\left(\sum_{m=1}^{M} m^2\right)\beta + \left(\sum_{m=1}^{M} m\right)\gamma = \sum_{m=1}^{M} m\ln r(m), \quad \text{(A7)}$$

$$\left(\sum_{m=1}^{M} m\right)\beta + \left(\sum_{m=1}^{M} 1\right)\gamma = \sum_{m=1}^{M} \ln r(m), \quad \text{(A8)}$$

the LS estimates are (the solution to these equations)

$$\begin{bmatrix} \hat{\beta} \\ \hat{\gamma} \end{bmatrix} = \begin{bmatrix} \sum_{m=1}^{M} m^2 & \sum_{m=1}^{M} m \\ \sum_{m=1}^{M} m & \sum_{m=1}^{M} 1 \end{bmatrix}^{-1} \begin{bmatrix} \sum_{m=1}^{M} m \ln r(m) \\ \sum_{m=1}^{M} \ln r(m) \end{bmatrix}. \tag{A9}$$

REFERENCES

[1] Y. Bar-Shalom, X. R. Li, and T. Kirubarajan
Estimation with Applications to Tracking and Navigation. Hoboken, NJ, USA: Wiley, 2001.

[2] Y. Bar-Shalom, P. K. Willett, and X. Tian
Tracking and Data Fusion. Storrs, CT, USA: YBS Publishing, 2011.

[3] Y. Bar-Shalom
"Airborne GMTI radar position bias estimation using static-rotator targets of opportunity,"
IEEE Trans. Aerosp. Electron. Syst., vol. 37, no. 2, pp. 695–699, Apr. 2001.

[4] P. D. Burns and W. D. Blair
"Sensor bias estimation from measurements of known trajectories,"
in *Proc. 37th Southeastern Symp. Syst. Theory*, Tuskegee, AL, USA, 2005.

[5] P. Braca, L. Millefiori, K. Bryan, and P. Willett
"Stochastic mean-reverting process for long-term prediction,"
IEEE Trans. Aerosp. Electron. Syst., vol. 52, no. 5, pp. 2313–2330, Sep. 2016.

[6] J. C. G. Franco
"Maximum-likelihood estimation of mean reverting processes," Onward, Inc., Working paper, 2003.

[7] K. Kastella and B. Yeary
"Bias modeling and estimation for GMTI applications,"
in *Proc. 3rd Intl. Conf. Inf. Fusion*, Paris, France, Jul. 2000.

[8] M. Kowalski, Y. Bar-Shalom, P. Willett, D. Belfadel, and F. Daum
"Bias CRLB in sine space for a 3-dimensional sensor,"
IEEE Trans. Aerosp. Electron. Syst., vol. 56, no. 1, pp. 673–686, Feb. 2020.

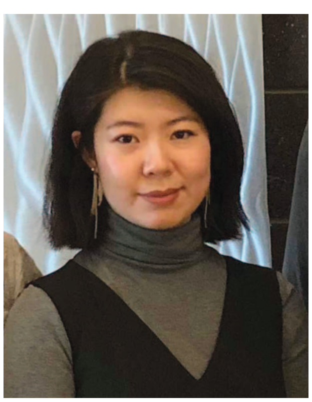
[9] T. Kailath
Linear Systems. Englewood Cliffs, NJ, USA: Prentice-Hall, 1980.

[10] X. Lin, Y. Bar-Shalom, and T. Kirubarajan
"Exact multi-sensor dynamic bias estimation with local tracks,"
IEEE Trans. Aerosp. Electron. Syst., vol. 40, no. 2, pp. 576–590, Apr.2004.

[11] X. Lin and Y. Bar-Shalom
"Multisensor target tracking performance with bias compensations,"
IEEE Trans. Aerosp. Electron. Syst., vol. 42, no. 3, pp. 1139–1149, Jul. 2006.

[12] L. M. Millefiori, P. Braca, K. Bryan, and P. Willett
"Modeling vessel kinematics using a stochastic mean-reverting process for long-term prediction,"
IEEE Trans. Aerosp. Electron. Syst., vol. 52, no. 5, pp. 2313–2330, Oct. 2016.

[13] L. M. Millefiori, P. Braca, and P. Willett
"Consistent estimation of randomly sampled Ornstein–Uhlenbeck process long-run mean for long-term target state prediction",
IEEE Signal Process. Lett., vol. 23, no. 11, pp. 1562–1566, Nov. 2016.



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