Heterogeneous and Asynchronous Information Matrix Fusion

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The Information Matrix Fusion (IMF) algorithm for nonlinear, asynchronous (with arbitrary local tracker sampling times for full rate as well as reduced-rate communication) and heterogeneous systems is presented. The heterogeneous estimates from local trackers are in different state spaces with different dimensions and are related by a nonlinear and noninvertible transformation. The main application of these results is the fusion of tracks from radar and infrared/electrooptical sensors. Different from Track-to-Track Fusion, the IMF does not require the cross-covariance between the local estimation errors. The performance of the proposed algorithm is shown via simulation based on Monte Carlo runs and is compared with the optimal solution—full-rate centralized fusion for both full-rate fusion and reduced-rate fusion for heterogeneous and asynchronous sensors.

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I. INTRODUCTION

A sensor configuration with complementary sensors at different locations (sensor network) is required in most of the tracking systems to achieve the necessary dependability and estimation accuracy. The best target-state-estimation performance is obtained by a centralized tracker/fuser (CTF), by directly sending to the fusion center (FC) all the measurements of the local sensors. However, CTF is not always available due to the communication constraints in practical situations. In this case, local sensors are capable of performing target-state tracking with their information processing systems. Such a system has a number of tracks that are sent to the FC. High-level algorithms such as Track-to-Track Fusion (T2TF) and Information Matrix Fusion (IMF) are commonly used for their modularity, practicality and scalability.

The IMF algorithm, as derived in [4] and originally presented in [6] for the case where all the local sensors are synchronized, is restricted to (i) linear systems, and (ii) the local trackers (LTs) estimate the same state. This algorithm belongs to the class of Track-to-Track Fusion with Memory (T2TFwM) and is, if operating at a "full communication rate," algebraically equivalent to the Configuration IV (centralized) tracker for linear systems. Unlike the Track-to-Track Fusion without Memory (T2TFwoM) [4], the IMF algorithm does not need the cross-covariance between the track estimation errors.

Asynchronous fusion was considered in [9] for the case where the local trackers estimate the same state. Aeberhard et al. [2] considered an asynchronous IMF with applications for driver-assistance systems. Heterogeneous fusion was investigated in [10], where it was shown how the linear minimum mean-square error (LMMSE) estimator and an (approximate) equivalent measurement based on a lower dimension local-state estimate can be used to update a higher dimension state estimate when these states are related by a nonlinear transformation. However, the fusion algorithm from [10] did not account for the common process noise. A modification of the result of [10] was given in [1] by using the unscented transform to evaluate the necessary covariances. The recent work of Mallick et al. [7] considered the problem of track fusion from heterogeneous sensors (with sampling intervals of the radar a multiple of the interval of the infrared/electrooptical (IR/EO) sensor) by augmenting the lower dimensional state of the IR/EO sensor with a number of range estimates based on a priori information, thus making it of the same dimension as the radar's state estimate. The work [11] derived the relationship between the process noise covariance matrices of the two state vectors (to account for the common process noise) and provided the expression of the covariance matrix of the heterogeneous estimation errors, which is needed in T2TF.

In [12], the IMF algorithm was first derived for nonlinear systems where the local filters are extended

Kalman filters (EKFs), rather than Kalman filters (KFs), estimating the same state and the conditions under which it holds. The fusion equations for the asynchronous case—arbitrary LT sensor sampling times—with LT-driven communication as well as FC-driven communication are also given for a system with homogeneous sensors.

In the present work, which is an extension of [12], the IMF algorithm is generalized to asynchronous heterogeneous systems where the local filters estimate different states (in different spaces of different dimensions), related by a nonlinear transformation, and the local sensors are running at different sampling rates. Such a situation occurs when the first tracker, using radar data, estimates the full Cartesian state of the target, while the second tracker, using a passive sensor (e.g., IR/EO), estimates the (lower dimension) angular state of the target. These two different-dimension states are related by a nonlinear transformation with no inverse. It is shown that one can combine state estimates of different dimensions in the IMF algorithm by constructing a "mapped" information involving the Jacobian of the nonlinear transformation that relates these states. Specifically, the IMF algorithm for an asynchronous case is derived for both an LT (full-rate) driven case and an FC (reduced-rate) driven case. This is investigated since the asynchronous case is motivated by the real-world scenario where the sensors have different sampling frequencies. In this work, we considered one-way communication from LT to FC without feedback. Note that there is no communication delay. In other words, the time index used later in the IMF formulas represents the LT sensor observation time, LT update time, as well as FC fusion time for the synchronous case. For the asynchronous case, zero communication delay implies that there is no out-of-order information received at FC from LT.

Section II presents the IMF algorithm for nonlinear filters with homogeneous sensors, i.e., the state estimates are in the same state space. Section III introduces the heterogeneous system in detail. The IMF algorithm for synchronous heterogeneous sensors is shown in Section IV . The asynchronous IMF for heterogeneous sensors is discussed in Section V for both the LT-driven case and the FC-driven case. Section VI presents the simulation results of an asynchronous heterogeneous IMF and compares them with the (i) optimal solution—centralized tracking/fusion and (ii) T2TF solution. Conclusions are presented in Section VII. Notations used in equations are summarized in Table I.

Table I
Notations Used for IMF Algorithm

Indices:	
t_k	Times when the FC carries out fusion
k	Discrete time index
T	Sampling interval
i	Sensor index
$t^r(t_k)$	Most recent times prior to t_k at which LT r sent information to the FC
$t^e(t_k)$	Most recent times prior to t_k at which LT e sent information to the FC
t^e_m	Sampling time of EO sensor with index m
t^r_l	Sampling time of radar with index l
Parameters:	
n^e	Dimension of state estimate from EO tracker
n^r	Dimension of state estimate from radar tracker
N_s	Number of local sensors
Variables:	
z^i	Measurement from sensor i
$\hat{x}^i$	State estimate of track i, used in homogeneous IMF
P^i	Covariance corresponding to $\hat{x}^i$, used in homogeneous IMF
$\hat{x}^e$, P^e	Estimate from EO sensor with dimension n^e and corresponding covariance
$\hat{x}^r$, P^r	Estimate from radar with dimension n^r and corresponding covariance
$\hat{x}^E$, P^E	Estimate obtained using the $\hat{x}^e$ with dimension n^r and corresponding covariance
$\hat{y}^E(k\|k)$	Mapped information state
$\hat{y}^E(k\|k-1)$	Mapped predicated information state

II. NONLINEAR INFORMATION MATRIX FUSION

The IMF with nonlinear filters for homogeneous sensors, i.e., the states are in the same space, is discussed in detail in [12] for both the synchronous case and the asynchronous case. The derivations will not be repeated here for the sake of brevity. In this section, it is assumed that each local filter/LT uses the same target state model with an EKF as the tracker, and they are synchronized.

Under full-rate communication, each LT communicates to the FC its updates as they are obtained and the FC then updates its fused state.

The LT-state update at sensor i at time t_k (indicated in the sequel by its index only) is given by [4]

$$\hat{x}^i(k|k) = \hat{x}^i(k|k-1) + P^i(k|k)H^i[k, \hat{x}^i(k|k-1)]^{\mathrm{T}} \cdot R^i(k)^{-1}\left[z^i(k) - h^i[k, \hat{x}^i(k|k-1)]\right] \tag{1}$$

using the measurements

$$z^i(k) = h^i[k, x(k)] + w^i(k), \tag{2}$$

where $w^i(k)$ is the zero-mean white measurement noise with covariance $R^i(k)$ and h^i is its measurement function, with Jacobian

$$H^i[k, \hat{x}^i(k|k-1)] \triangleq \left[\nabla_{x(k)} h^i[k, x(k)]^{\mathrm{T}}\right]^{\mathrm{T}}\Big|_{x(k)=\hat{x}^i(k|k-1)}. \tag{3}$$

The covariance-update equation in the information matrix form is (see [3, eq. (5.2.3-16)])

$$P^i(k|k)^{-1} = P^i(k|k-1)^{-1} + H^i[k, \hat{x}^i(k|k-1)]^{\mathrm{T}} \cdot R^i(k)^{-1}H^i[k, \hat{x}^i(k|k-1)]. \tag{4}$$

The EKF information-state update at sensor i is [11]

$$\begin{aligned} &P^i(k|k)^{-1}\hat{x}^i(k|k) \\ &= P^i(k|k-1)^{-1}\hat{x}^i(k|k-1) + H^i[k, \hat{x}^i(k|k-1)]^{\mathrm{T}} \\ &\quad \cdot R^i(k)^{-1}\left[z^i(k) - h^i[k, \hat{x}^i(k|k-1)]\right] \\ &\quad + H^i[k, \hat{x}^i(k|k-1)]\hat{x}^i(k|k-1), \end{aligned} \tag{5}$$

which gives (by rearranging)[1]

$$\begin{aligned} &H^i[k, \hat{x}^i(k|k-1)]^{\mathrm{T}} R^i(k)^{-1}\left[z^i(k) - h^i[k, \hat{x}^i(k|k-1)]\right] \\ &\quad + H^i[k, \hat{x}^i(k|k-1)]\hat{x}^i(k|k-1) \\ &\quad = P^i(k|k)^{-1}\hat{x}^i(k|k) - P^i(k|k-1)^{-1}\hat{x}^i(k|k-1). \end{aligned} \tag{6}$$

The *full-rate information-state fusion* equation is

$$\begin{aligned} P(k|k)^{-1}\hat{x}(k|k) &= P(k|k-1)^{-1}\hat{x}(k|k-1) \\ &\quad + \sum_{i=1}^{N_s}\left[P^i(k|k)^{-1}\hat{x}^i(k|k)\right. \\ &\quad \left. - P^i(k|k-1)^{-1}\hat{x}^i(k|k-1)\right], \end{aligned} \tag{7}$$

where N_s is the number of sensors. The differences of the predicted and updated information states in the summation above are the "new information" from each of the sensors. This new information is exactly equivalent to the innovation in the KF in the linear case—see (6)—and, thus, it is uncorrelated with the past information.

The *information matrix fusion* equation is

$$P(k|k)^{-1} = P(k|k-1)^{-1} + \sum_{i=1}^{N_s}[P^i(k|k)^{-1} - P^i(k|k-1)^{-1}], \tag{8}$$

i.e., the same as in the linear case but subject to the approximations (linearization).

III. HETEROGENEOUS STATES

The IMF algorithm, as shown in (7), requires the addition of the information vectors $(P^i)^{-1}\hat{x}^i$ across the local trackers, i.e., they have to have the same dimension. In the case where one sensor is a radar (with the corresponding target estimate of dimension n^r and its covariance, superscripted by r) and the other sensor is an IR/EO one (with the corresponding target estimate of dimension n^e and its covariance, superscripted by e), the two estimated state vectors have different dimensions (they are in different spaces). Consequently, the corresponding new information, based on the different-dimension local information states, cannot be added as required by (7).

The smaller dimension (n^e) state is related to the higher dimension (n^r) state according to

$$x^e = g(x^r, p^r, p^e), \tag{9}$$

where the time arguments are omitted for simplicity, and p^r and p^e are the position vectors of the radar and the EO sensor, respectively. Since $g(\cdot)$ maps the n^r-dimensional vector x^r to the (lower) n^e-dimensional vector x^e, it is not invertible, i.e., one cannot obtain an estimate of the full Cartesian state (of dimension n^r) based on the angular state estimate from the EO sensor.

Consider the estimate $\hat{x}^e$ from the EO sensor (local track) as an "observation" z of the n^r-dimensional state vector x^r (truth) related by

$$z = g(x^r) + w, \tag{10}$$

where w is a zero-mean noise with covariance matrix P^e. The solution (desired estimate) of the above system is defined as $\hat{x}^E$ (of dimension n^r), where the superscript E indicates the estimate obtained using the observation $\hat{x}^e$ through the nonlinear relationship $g(\cdot)$.[2] The (hypothetical[3]) least-squares (LS) estimate of dimension n^r based on (10) (following [3, eq. (3.4.4-11)], using the radar estimate $\hat{x}^r$ since the true state is not available, is given by

$$\hat{x}^E = (G'R^{-1}G)^{-1}G'R^{-1}(z - h(\hat{x}^r)) + \hat{x}^r \tag{11}$$

and the covariance corresponding to $\hat{x}^E$ is

$$P^E = (G'R^{-1}G)^{-1}, \tag{12}$$

where G is the Jacobian evaluated at $\hat{x}^r$ and

$$G(\hat{x}^r) \triangleq \left[\nabla_{x^r} g[x^r]^{\mathrm{T}}\right]^{\mathrm{T}}\Big|_{x^r=\hat{x}^r} \tag{13}$$

is the $(n^e \times n^r)$ Jacobian. Note that $G'R^{-1}G$ is not invertible because the right-hand side of (12) has rank $n^e < n^r$, and the covariance matrix does not exist. In this case, one cannot obtain an estimate of the full Cartesian state (of dimension n^r) based on the angular-state estimate from the EO sensor. However, the above equations provide the motivation for the following implementable algorithm that can overcome the unequal-state-dimension problem: Instead of the "mapped" estimate (11) and covariance matrix (12), one can calculate the "mapped new information" directly, which is what the IMF equations need. The following approach is taken to overcome the incompatibility of state dimensions and singularity of (12).

IV. HETEROGENEOUS IMF FOR SYNCHRONOUS CASE

Define an n^r-dimensional "mapped (from the EO/IR state space to radar state space) information state"

$$\hat{y}^E(k|k) \triangleq P^E(k|k)^{-1}\hat{x}^E(k|k) \tag{14}$$

and a "mapped predicted information state"

$$\hat{y}^E(k|k-1) \triangleq P^E(k|k-1)^{-1}\hat{x}^E(k|k-1) \tag{15}$$

[1]Note that equations (5) and (6) in [12] have typos and the correct ones are, respectively, given in (5) and (6) in this paper.

[2]This is (9) omitting the sensor positions for simplicity.

[3]To be defined in the sequel—this estimate cannot be obtained since $n^e < n^r$.

in order to obtain the "mapped new information" $\hat{y}^E(k|k) - \hat{y}^E(k|k-1)$. Although $\hat{y}^E(k|k)$ and $\hat{y}^E(k|k-1)$ cannot be obtained since $\hat{x}^E$ and P^E are not available, they are not necessary and *only their difference*—the "mapped new information"—will be needed in the information-state-fusion equation, as it will be shown in the sequel.

The state update equation for (10) with the time arguments, based on (1), can be written as

$$\hat{x}^E(k|k) = \hat{x}^E(k|k-1) + P^E(k|k)G[k, \hat{x}^E(k|k-1)]^{\mathrm{T}} \cdot P^e(k|k)^{-1}\left[\hat{x}^e(k|k) - g[k, \hat{x}^E(k|k-1)]\right], \tag{16}$$

where

$$G[k, \hat{x}^E(k|k-1)] \triangleq \left[\nabla_x g[x]^{\mathrm{T}}\right]^{\mathrm{T}}\Big|_{x=\hat{x}^E(k|k-1)} \tag{17}$$

is the $(n^e \times n^r)$ Jacobian, the evaluation of which is discussed in the sequel. The covariance update equation in the information matrix form, based on (4), is

$$P^E(k|k)^{-1} = P^E(k|k-1)^{-1} + G[k, \hat{x}^E(k|k-1)]^{\mathrm{T}} \cdot P^e(k|k)^{-1}G[k, \hat{x}^E(k|k-1)]. \tag{18}$$

The "mapped new information" from the sensor of dimension n^e into the space of dimension $n^r > n^e$ can be obtained, by substituting (16) and (18) into (14) and (15) and following (6), as[4]

$$\begin{aligned}
&\hat{y}^E(k|k) - \hat{y}^E(k|k-1) \\
&= G[k, \hat{x}^E(k|k-1)]^{\mathrm{T}}P^e(k|k)^{-1} \\
&\quad \cdot \left\{\hat{x}^e(k|k) - g[k, \hat{x}^E(k|k-1)]\right. \\
&\quad \left. +\ G[k, \hat{x}^E(k|k-1)]\hat{x}^E(k|k-1)\right\}
\end{aligned} \tag{19}$$

$$\begin{aligned}
&\approx G[k, \hat{x}(k|k-1)]^{\mathrm{T}}P^e(k|k)^{-1} \\
&\quad \left\{\hat{x}^e(k|k) - g[k, \hat{x}(k|k-1)]\right. \\
&\quad \left. +\ G[k, \hat{x}(k|k-1)]\hat{x}(k|k-1)\right\},
\end{aligned} \tag{20}$$

where the approximate equality above is obtained by the substitution $\hat{x}^E \to \hat{x}$. The evaluation of the Jacobian (17) and state prediction in (19) can be done using the FC estimate $\hat{x}$ rather than $\hat{x}^E$ (since the latter is not available). In this case, we use $G[k, \hat{x}(k|k-1)]$ and $g[k, \hat{x}(k|k-1)]$ instead of $G[k, \hat{x}^E(k|k-1)]$ and $g[k, \hat{x}^E(k|k-1)]$, respectively. Therefore, (20) will be used to obtain the *synchronous heterogeneous information state fusion equation* with full-rate communication by modifying (7) as follows:

$$\begin{aligned}
&P(k|k)^{-1}\hat{x}(k|k) \\
&= P(k|k-1)^{-1}\hat{x}(k|k-1) \\
&\quad +\left\{P^r(k|k)^{-1}\hat{x}^r(k|k) - P^r(k|k-1)^{-1}\hat{x}^r(k|k-1)\right\} \\
&\quad +\left\{\hat{y}^E(k|k) - \hat{y}^E(k|k-1)\right\}.
\end{aligned} \tag{21}$$

Note that the entire right-hand side of (21) has dimension n^r, i.e., the problem of unequal state dimensions has been eliminated. The corresponding *synchronous heterogeneous information matrix fusion equation*, based on (18) and (8), is

$$\begin{aligned}
&P(k|k)^{-1} \\
&= P(k|k-1)^{-1} + \left\{P^r(k|k)^{-1} - P^r(k|k-1)^{-1}\right\} \\
&\quad +\left\{P^E(k|k)^{-1} - P^E(k|k-1)^{-1}\right\} \\
&= P(k|k-1)^{-1} + \left\{P^r(k|k)^{-1} - P^r(k|k-1)^{-1}\right\} \\
&\quad +\left\{G[k, \hat{x}^E(k|k-1)]^{\mathrm{T}}P^e(k|k)^{-1}G[k, \hat{x}^E(k|k-1)]\right\}.
\end{aligned} \tag{22}$$

At initialization, one needs the radar's (full-state) estimate to evaluate the Jacobian G.

The fusion architecture for a synchronous heterogeneous IMF is shown in Fig. 1, where the dashed circle indicates the mapping of the new information from angle space to Cartesian space.

V. HETEROGENEOUS IMF FOR ASYNCHRONOUS CASE

A. LT-Driven Asynchronous Case

With LT/local filter-driven communication, the fusion in an asynchronous system (e.g., with tracks from radar and IR/EO sensors) is carried out whenever the FC receives new information. In this case, the system is updated with full rate. As shown in Fig. 2, sensor r is assumed to be the active one (radar) with the state vector in the larger state space (of dimension $n^r_{\mathbf{x}}$) and sensor e is the passive EO/IR with the state vector in the smaller state space (of dimension $n^e_{\mathbf{x}} < n^r_{\mathbf{x}}$). For the FC, the fusion times are equal to the times when new information is obtained. From Fig. 2, we have

$$t_k = t^e_m \tag{23}$$

and, with $l \ni t^r_l < t_k$,

$$t_{k-1} = \max\{t^r_l, t^e_{m-1}\}, \tag{24}$$

where l and m denote the respective LT sampling indices. LT-driven asynchronous fusion (full rate) is carried out whenever an LT has new information delivered to the FC.

The "mapped new information," based on (20), is

$$\begin{aligned}
&\hat{y}^E(t_k|t_k) - \hat{y}^E[t_k|t^e(t_k)] \\
&= G\big[t_k, \hat{x}^E[t_k|t^e(t_k)]\big]^{\mathrm{T}}P^e(t_k|t_k)^{-1} \\
&\quad \cdot \left\{\hat{x}^e(t_k|t_k) - g\big[t_k, \hat{x}^E[t_k|t^e(t_k)]\big]\right. \\
&\quad \left. +\ G\big[t_k, \hat{x}^E[t_k|t^e(t_k)]\big]\hat{x}^E[t_k|t^e(t_k)]\right\}.
\end{aligned} \tag{25}$$

[4] In (6), we substitute $H^i \to G, R^i \to P^e, z^i \to \hat{x}^e, h^i \to g$, and $\hat{x}^i \to \hat{x}^E$.

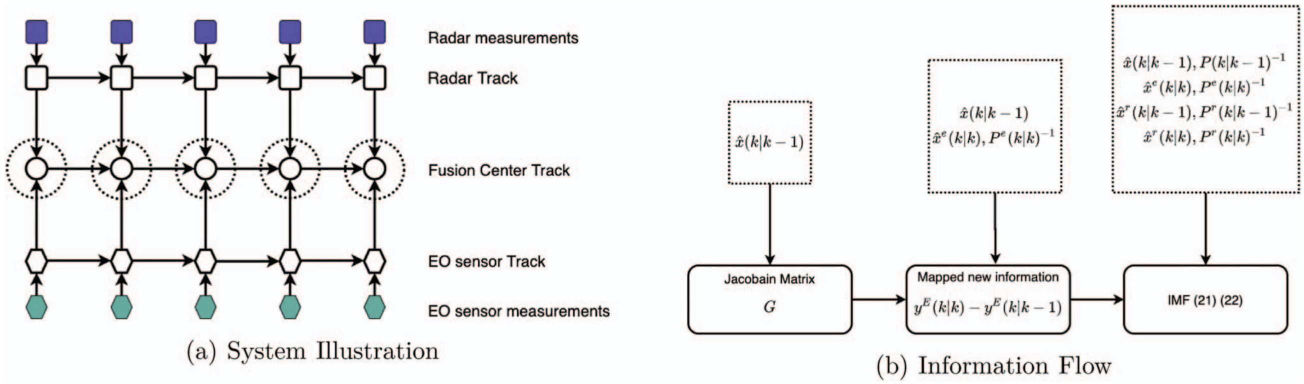

Fig. 1. Fusion architecture for synchronous heterogeneous IMF.

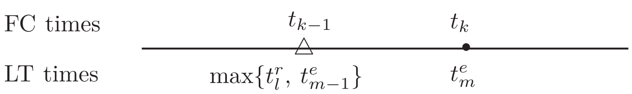

Fig. 2. FC times and LT times in asynchronous IMF—LT driven, update with latest information from sensor e.

Based on the above and discussion about the asynchronous but homogeneous system in [12], the asynchronous heterogeneous information state fusion equation becomes

$$\begin{aligned}
&P(t_k|t_k)^{-1}\hat{x}(t_k|t_k) \\
&\quad = P(t_k|t_{k-1})^{-1}\hat{x}(t_k|t_{k-1}) \\
&\qquad + \{P^r(t_k|t_k)^{-1}\hat{x}^r(t_k|t_k) \\
&\qquad - P^r[t_k|t^r(t_k)]^{-1}\hat{x}^r[t_k|t^r(t_k)]\}\chi^r(t_k) \\
&\qquad + \{\hat{y}^E(t_k|t_k) - \hat{y}^E[t_k|t^e(t_k)]\}\chi^e(t_k),
\end{aligned} \tag{26}$$

where $t^r(t_k)$ and $t^e(t_k)$ are the most recent times prior to t_k at which LT r and LT e have sent information to the FC (its previous communication), respectively, and $\chi^r(k)$ and $\chi^e(k)$ are the communication indicator functions for LTs,

$$\chi^r(k) = \begin{cases} 1 & \text{if LT } r \text{ sends information to FC at } t_k \\ 0 & \text{otherwise} \end{cases} \tag{27}$$

and

$$\chi^e(k) = \begin{cases} 1 & \text{if LT } e \text{ sends information to FC at } t_k \\ 0 & \text{otherwise} \end{cases}. \tag{28}$$

Note that if $\chi^e(t_k) = 1$ and $\chi^r(t_k) = 0$, (26) carries out the update with the latest information only from sensor e, as illustrated in Fig. 2. If $\chi^e(t_k) = 0$ and $\chi^r(t_k) = 1$, then (26) carries out the update with the latest information only from sensor r.

The corresponding information matrix fusion equation is (modifying (22)) given by

$$\begin{aligned}
P(t_k|t_k)^{-1} = P(t_k|t_{k-1})^{-1} &+ \{P^r(t_k|t_k)^{-1} - P^r[t_k|t^r(t_k)]^{-1}\}\chi^r(t_k) \\
&+ \{G[t_k, \hat{x}^E[t_k|t^e(t_k)]]^{\mathrm{T}} P^e(t_k|t_k)^{-1} \\
&\cdot G[t_k, \hat{x}^E[t_k|t^e(t_k)]]\}\chi^e(t_k).
\end{aligned} \tag{29}$$

To implement the above method, the approximations

$$G[t_k, \hat{x}^E[t_k|t^e(t_k)]] \approx G[t_k, \hat{x}[t_k|t^e(t_k)]] \tag{30}$$

and

$$\hat{x}^E[t_k|t^e(t_k)] \approx \hat{x}[t_k|t^e(t_k)] \tag{31}$$

are used in (25), (26), and (29).

The fusion architecture for an LT-driven asynchronous heterogeneous IMF is shown in Fig. 3, where the dashed circle indicates the mapping of the new information from angle space to Cartesian (25).

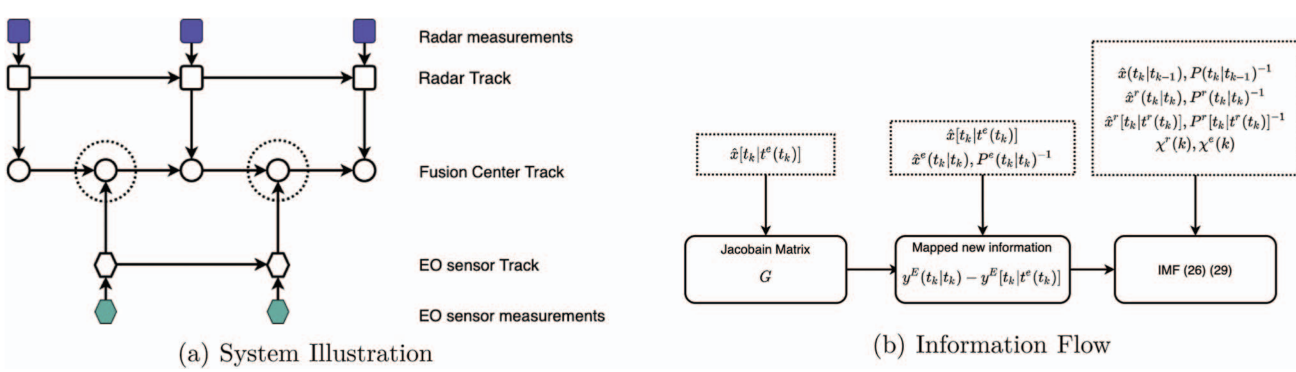

Fig. 3. Fusion architecture for LT-driven asynchronous heterogeneous IMF.

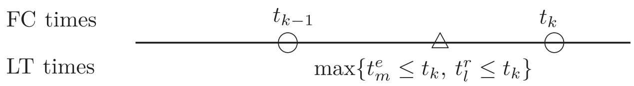

Fig. 4. FC times and LT times in asynchronous IMF—FC driven.

B. FC-Driven Asynchronous Case

In this case, it is assumed that the FC updates its state at intervals τ_k (length of the fusion window); namely, if the current update is at t_k, the previous update was at $t_k - \tau_k = t_{k-1}$. The fusion window ending at t_k is thus the semiclosed interval $(t_k - \tau_k, t_k]$ of length τ_k, which is at the discretion of the FC. The system is updated with a reduced rate. The fusion times at the FC and the latest LT sampling times are shown in Fig. 4.

The information state fusion with FC-driven communication is

$$
\begin{aligned}
&P(t_k|t_k)^{-1}\hat{x}(t_k|t_k) \\
&\quad = P(t_k|t_{k-1})^{-1}\hat{x}(t_k|t_{k-1}) \\
&\qquad + \{P^r[t_k|t^r(t_k)]^{-1}\hat{x}^r[t_k|t^r(t_k)] \\
&\qquad - P^r[t_k|t^r(t_{k-1})]^{-1}\hat{x}^r[t_k|t^r(t_{k-1})]\} \\
&\qquad + \{\hat{y}^E[t_k|t^e(t_k)] - \hat{y}^E[t_k|t^e(t_{k-1})]\}
\end{aligned}
\tag{32}
$$

and the counterpart of (25) is

$$
\begin{aligned}
&\hat{y}^E[t_k|t^e(t_k)] - \hat{y}^E[t_k|t^e(t_{k-1})] \\
&\quad = G\big[t_k, \hat{x}^E[t_k|t^e(t_{k-1})]\big]^{\mathrm{T}} P^e[t_k|t^e(t_k)]^{-1} \\
&\qquad \cdot \{\hat{x}^e[t_k|t^e(t_k)] - g\big[t_k, \hat{x}^E[t_k|t^e(t_{k-1})]\big] \\
&\qquad + G\big[t_k, \hat{x}^E[t_k|t^e(t_{k-1})]\big]\hat{x}^E[t_k|t^e(t_{k-1})]\}
\end{aligned}
\tag{33}
$$

$$
\begin{aligned}
&\approx G\big[t_k, \hat{x}[t_k|t^e(t_{k-1})]\big]^{\mathrm{T}} P^e[t_k|t^e(t_k)]^{-1} \\
&\quad \cdot \{\hat{x}^e[t_k|t^e(t_k)] - g\big[t_k, \hat{x}[t_k|t^e(t_{k-1})]\big] \\
&\quad + G\big[t_k, \hat{x}[t_k|t^e(t_{k-1})]\big]\hat{x}[t_k|t^e(t_{k-1})]\},
\end{aligned}
\tag{34}
$$

where $t^r(t_k)$, $t^e(t_k)$ and $t^r(t_{k-1})$, $t^e(t_{k-1})$ are the times of the most recent update of LT r and e prior to t_k and t_{k-1}, respectively. The approximation[5] in (39) is needed to evaluate the Jacobian matrix since $\hat{x}^E$ is not available.

The terms in the braces in (32) represent the accumulated new information from sensors r and e during the fusion window $(t_{k-1}, t_k]$ and are mapped directly to the fusion time t_k. Note that if the most recent update of LT r or LT e prior to t_k occurs prior to t_{k-1}, i.e., there is no new information from this LT during the window $(t_{k-1}, t_k]$, then the terms in the braces corresponding to each LT will be equal and thus cancel—the "new information" from this LT during this window is zero in this case.

The corresponding information matrix fusion equation is

$$
\begin{aligned}
&P(t_k|t_k)^{-1} \\
&\quad = P(t_k|t_{k-1})^{-1} + \{P^r[t_k|t^r(t_k)]^{-1} - P^r[t_k|t^r(t_{k-1})]^{-1}\} \\
&\qquad + \{G\big[t_k, \hat{x}^E[t_k|t^e(t_{k-1})]\big]^{\mathrm{T}} P^e[t_k|t^e(t_k)]^{-1} \\
&\qquad \cdot G\big[t_k, \hat{x}^E[t_k|t^e(t_{k-1})]\big]\}
\end{aligned}
\tag{35}
$$

$$
\begin{aligned}
&\approx P(t_k|t_{k-1})^{-1} + \{P^r[t_k|t^r(t_k)]^{-1} - P^r[t_k|t^r(t_{k-1})]^{-1}\} \\
&\quad + \{G\big[t_k, \hat{x}[t_k|t^e(t_{k-1})]\big]^{\mathrm{T}} P^e[t_k|t^e(t_k)]^{-1} \\
&\quad \cdot G\big[t_k, \hat{x}[t_k|t^e(t_{k-1})]\big]\}.
\end{aligned}
\tag{36}
$$

The "new information" terms in the braces in (36) are not uncorrelated from the past information even in the linear case—the uncorrelatedness holds only for full-rate communication. Their use for "decorrelation" from the past is only approximate.

The fusion architecture for an LT-driven asynchronous heterogeneous IMF is shown in Fig. 5, where the dashed circle indicates the mapping of the new information from angle space to Cartesian (39).

VI. SIMULATION RESULTS

The asynchronous heterogeneous IMF is evaluated for two cases for the scenario detailed in the sequel: (i) full-rate (LT-driven) asynchronous LTs and (ii) reduced-rate (FC-driven) asynchronous LTs. The performance of synchronous and heterogeneous LTs is also evaluated. The homogeneous and synchronous heterogeneous cases are discussed in [12] and will not be duplicated here.

A. The State Models for the Active and Passive Sensors

In the ξ–η space, a radar located at $[\xi^r\ \eta^r]$ with, for simplicity, direct Cartesian position measurements with measurement noises w^r [6] and an EO sensor located at $[\xi^e\ \eta^e]$ with bearing measurements only,

$$
\theta^e = \tan^{-1}[(\eta - \eta^e)/(\xi - \xi^e)] + w^e,
\tag{37}
$$

are considered for the IMF for a two-dimensional (2-D) target. The measurement noises w^r and w^e are assumed to be independent zero-mean white Gaussians with corresponding standard deviations σ^r and σ^e.

The active sensor (radar) provides 2-D measurements in 2-D Cartesian space (position) and a 4-D LT state (position and velocity) with a discretized continuous time white noise acceleration (CWNA) motion

[5] $G\big[t_k, \hat{x}^E[t_k|t^e(t_{k-1})]\big] \approx G\big[t_k, \hat{x}[t_k|t^e(t_{k-1})]\big]$, $\hat{x}^E[t_k|t^e(t_{k-1})] \approx \hat{x}[t_k|t^e(t_{k-1})]$.

[6] The radar's measurements in polar coordinates can be transformed into Cartesian coordinates with an unbiased consistent transformation [3, Ch. 10.4.3].

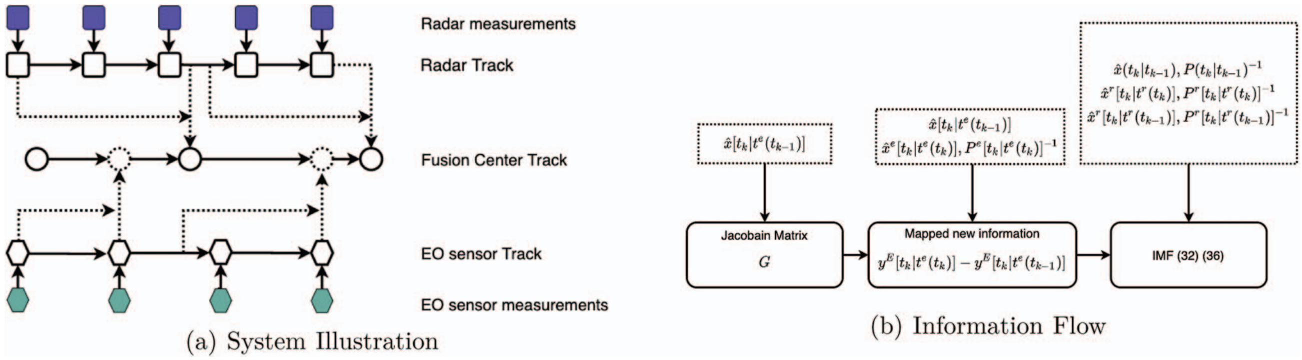

Fig. 5. Fusion architecture for FC-driven asynchronous heterogeneous IMF.

model [3] in Cartesian coordinates:

$$\mathbf{x}^r = [\xi \;\; \dot{\xi} \;\; \eta \;\; \dot{\eta}]^{\mathrm{T}} \tag{38}$$

with the discretized dynamic model to be

$$\mathbf{x}^r(t^r_{l+1}) = F^r \mathbf{x}^r(t^r_l) + \mathbf{v}^r(t^r_l), \tag{39}$$

and measurement model

$$\mathbf{z}^r = H^r \mathbf{x}^r(t^r_l) + \mathbf{w}^r(t^r_l), \tag{40}$$

where

$$F^r = \begin{bmatrix} 1 & T^r & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T^r \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{41}$$

$$H^r = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}. \tag{42}$$

The process noise vector has covariance matrix

$$Q^r = \begin{bmatrix} \frac{1}{3}(T^r)^3 & \frac{1}{2}(T^r)^2 & 0 & 0 \\ \frac{1}{2}(T^r)^2 & T^r & 0 & 0 \\ 0 & 0 & \frac{1}{3}(T^r)^3 & \frac{1}{2}(T^r)^2 \\ 0 & 0 & \frac{1}{2}(T^r)^2 & T^r \end{bmatrix} \tilde{q}, \tag{43}$$

where $\tilde{q}$ is the power spectral density and $\tilde{q} = 3.8\ \mathrm{m}^2/\mathrm{s}^3$ in simulations.

The EO sensor uses a KF also based on a CWNA model with a state vector involving the angle and angle rate

$$\mathbf{x}^e = [\theta \;\; \dot{\theta}]^{\mathrm{T}}. \tag{44}$$

The discretized dynamic model is

$$\mathbf{x}^e(t^e_m + 1) = F^e \mathbf{x}^e(t^e_m) + \mathbf{v}^e(t^e_m), \tag{45}$$

$$\mathbf{z}^e = H^e \mathbf{x}^e(t^e_m) + \mathbf{w}^e(t^e_m), \tag{46}$$

where

$$F^e = \begin{bmatrix} 1 & T^e \\ 0 & 1 \end{bmatrix}, \tag{47}$$

$$H^e = \begin{bmatrix} 1 & 0 \end{bmatrix}. \tag{48}$$

The state vector (38) and the state vector (44) have a nonlinear relationship

$$\mathbf{x}^e = \alpha\,[\mathbf{x}^r] \tag{49}$$

with explicit expressions

$$\theta = \mathrm{atan}\frac{\eta - \eta^e}{\xi - \xi^e}, \tag{50}$$

$$\dot{\theta} = \frac{v\sin(\phi)}{r^e}, \tag{51}$$

where v is the target speed given by

$$v = \sqrt{\dot{\xi}^2 + \dot{\eta}^2}, \tag{52}$$

r^e is the range with respect to the passive sensor's location given by

$$r^e = \sqrt{(\xi - \xi^e)^2 + (\eta - \eta^e)^2}, \tag{53}$$

and ϕ is the difference between velocity angle and position azimuth angle given by

$$\phi = \mathrm{atan}\frac{\dot{\eta}}{\dot{\xi}} - \mathrm{atan}\frac{\eta - \eta^e}{\xi - \xi^e}. \tag{54}$$

The process noise covariance matrix of the EO tracker's model at time t_k has the following relationship

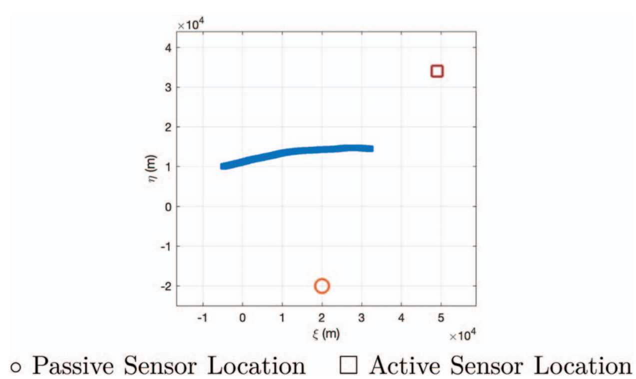

Fig. 6. Target trajectory (one realization) and sensor locations.

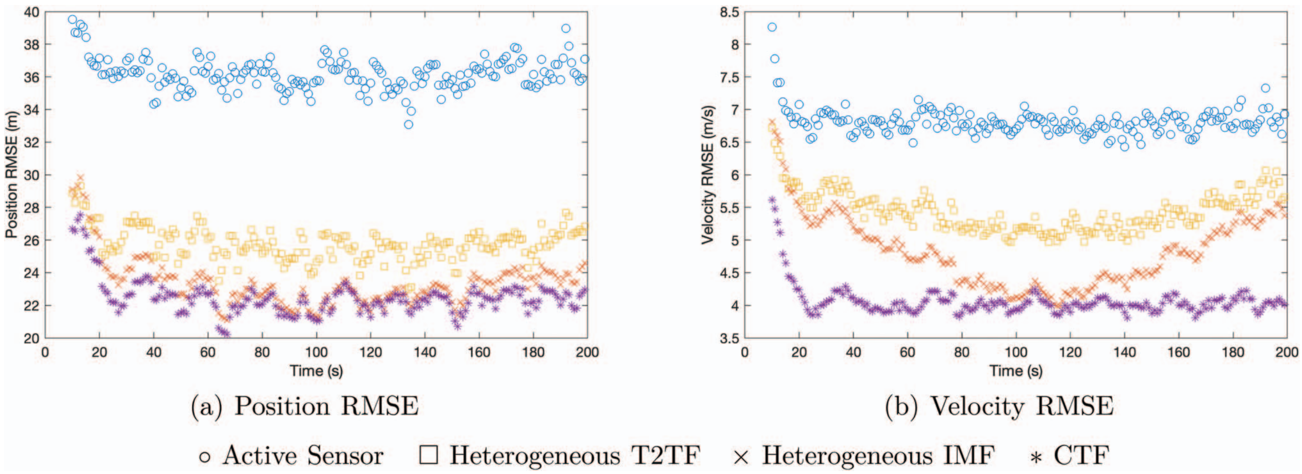

Fig. 7. Full-rate (LT-driven) heterogeneous IMF RMSE from 500 runs.

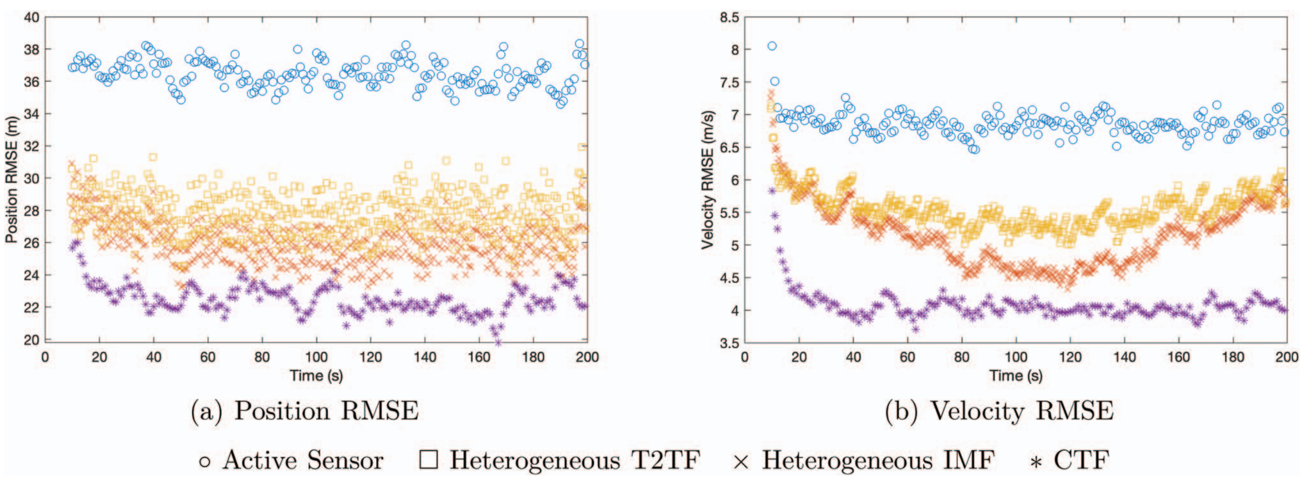

Fig. 8. Reduced-rate ($T^{\mathrm{FC}} = 0.4$ s) heterogeneous IMF RMSE from 500 runs.

with the active process noise covariance matrix, as derived and discussed in [12][7]:

$$Q^e(t_{k+1}, t_k) = A(t_k) Q^r(t_{k+1}, t_k) A(t_k)', \tag{55}$$

where

$$A(t_k) \triangleq \left[\nabla_x \alpha(x)^{\mathrm{T}} \right]^{\mathrm{T}} \Big|_{x = F^r[t_{k+1}, t_k] \mathbf{x}^r(t_k)}. \tag{56}$$

B. Numerical Results

The sensor locations are [49 34] km and [−20 20] km for the active sensor and passive sensor, respectively. The target is assumed to have an initial position [−5 10] km and velocity [200 20] m/s. The trajectory lasts for 200 s. Fig. 6 shows the target trajectory (one realization) and sensor locations. The standard deviations of measurement noises are assumed to be $\sigma^r = 50$ m for the active sensor (direct position measurement in both coordinates) and $\sigma^e = 0.4$ milliradian (mrad) for the passive sensor (azimuth angle). In all asynchronous cases, the active sensor (radar) has sampling interval $T^r = 1$ s and the passive sensor (EO) has sampling interval $T^e = 0.1$ s.

Several FC sampling intervals (fusion rates) are used in the simulation to compare the performance of the proposed algorithm. The simulation results are based on 500 Monte Carlo runs. To evaluate the performance of the IMF: (i) the full-rate centralized tracking/fusion is carried out, which is the optimal one can achieve and (2) the heterogeneous T2TF [13] is also carried out. Note that the RMSE results in Figs 7–10 started at 9 s after the convergence of LFs to avoid large plot scales.

The reduced-rate asynchronous heterogeneous IMF is evaluated with multiple sampling rates at the FC: $T_{\mathrm{FC}} = 0.4$, 0.8, and 1.6 s. The RMSEs for both position and velocity are evaluated. Fig. 7 shows the RMSE of the full-rate IMF. In this case, the rate is the that of a higher rate sensor (10 Hz, since $T^e = 0.1$ s). Simulation results for $T_{\mathrm{FC}} = 0.4$, 0.8, and 1.6 s are shown in Figs. 8, 9, and 10, respectively. The oscillations of the position errors of the IMF are due to the fact that at its update time it uses a predicted active LT (radar) state since the FC is not synchronized with the radar. It can be seen that with full-rate communication, the proposed

[7]This process noise covariance mapping is similar to [8] except for the linearization Jacobian, which is evaluated at the IMF fused state, while [8] used a "worst-case"-based mapping.

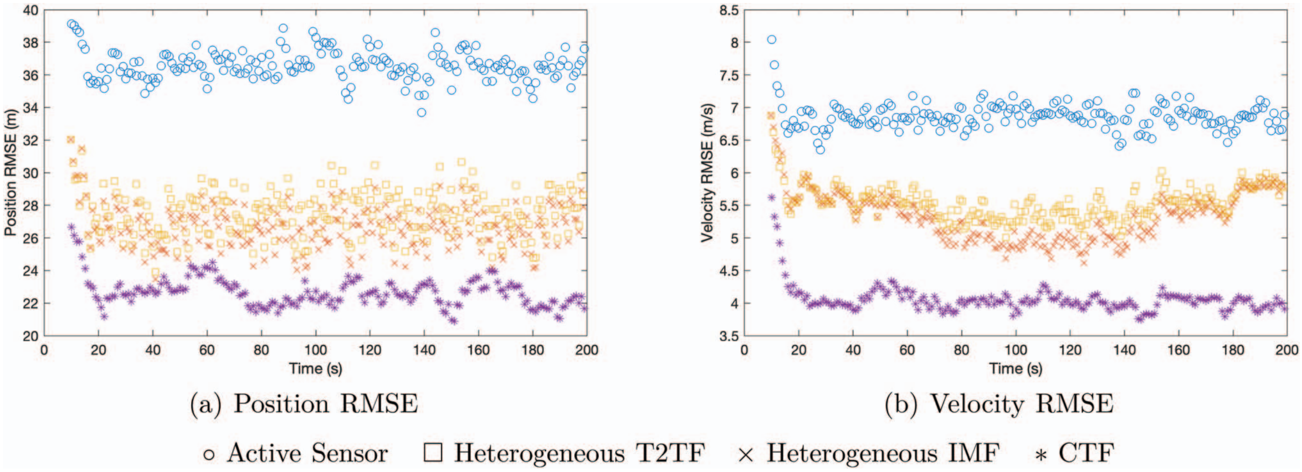

Fig. 9. Reduced-rate ($T^{\mathrm{FC}} = 0.8$ s) heterogeneous IMF RMSE from 500 runs.

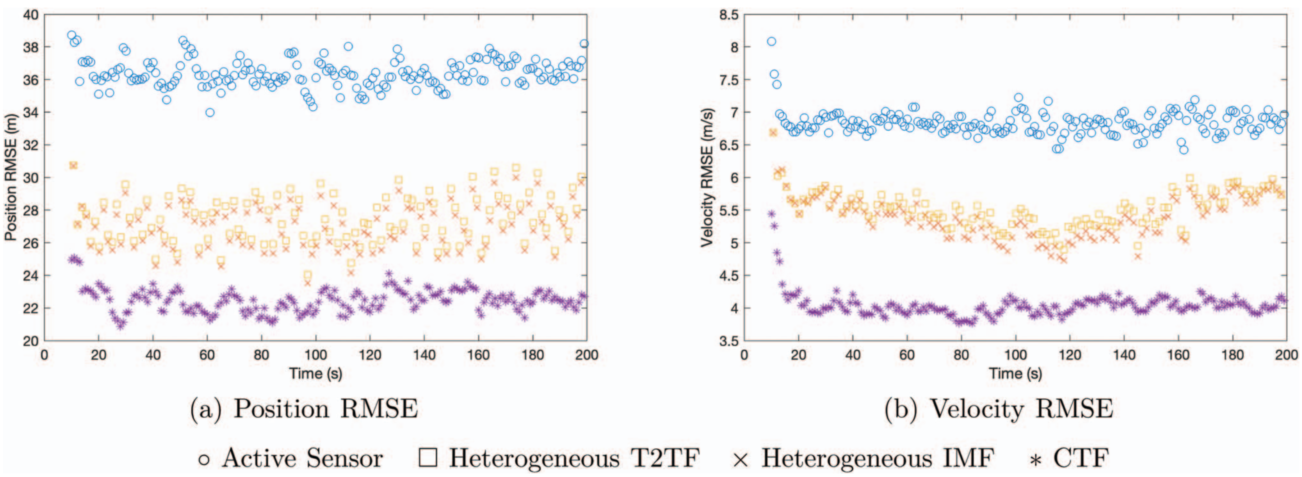

Fig. 10. Reduced-rate ($T^{\mathrm{FC}} = 1.6$ s) heterogeneous IMF RMSE from 500 runs.

heterogeneous and asynchronous IMF achieves almost the optimal result (the CTF result) for position. The performance of the velocity fusion is somewhat off due to the nonlinearity (linear motion in Cartesian space is not linear in angle space), and approximation of the "new information," The results also depend on the geometry between the sensors and the target trajectory. It can be seen that a larger sampling interval at the FC will degrade the performance of the IMF; however, there is always a reduction in the RMSE compared to the case with an active sensor only for both position and velocity. In all the cases considered, the proposed IMF has better performance than T2TF by having a smaller RMSE for both position and velocity.

VII. CONCLUSION

In this work, the IMF algorithm was extended to nonlinear, asynchronous, and heterogeneous systems. The LTs from an active sensor and a passive sensor are in different state spaces and are related by a nonlinear transformation without inverse. Both the LT-driven full-rate asynchronous case and FC-driven reduced-rate asynchronous case are investigated. Although the passive (EO/IR) LT state with a lower dimension cannot be used directly in the IMF, it has been shown that its new information can be mapped to the high-dimension state space and then used by the IMF at the FC. With full-rate communication (LT driven), the proposed IMF can almost achieve the optimal solution (full-rate CTF). The performance of the FC-driven asynchronous IMF is not optimal but still remarkable compared with the results from the active sensor (radar) only and T2TF by achieving a smaller RMSE in both position and velocity. Real data testing is not available at the current stage; however, it will be investigated in future works.

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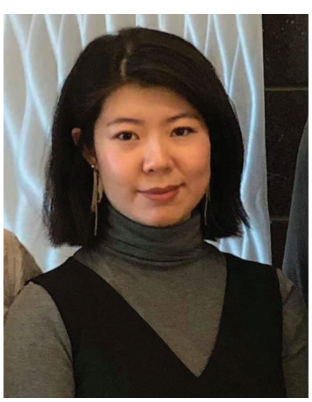

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